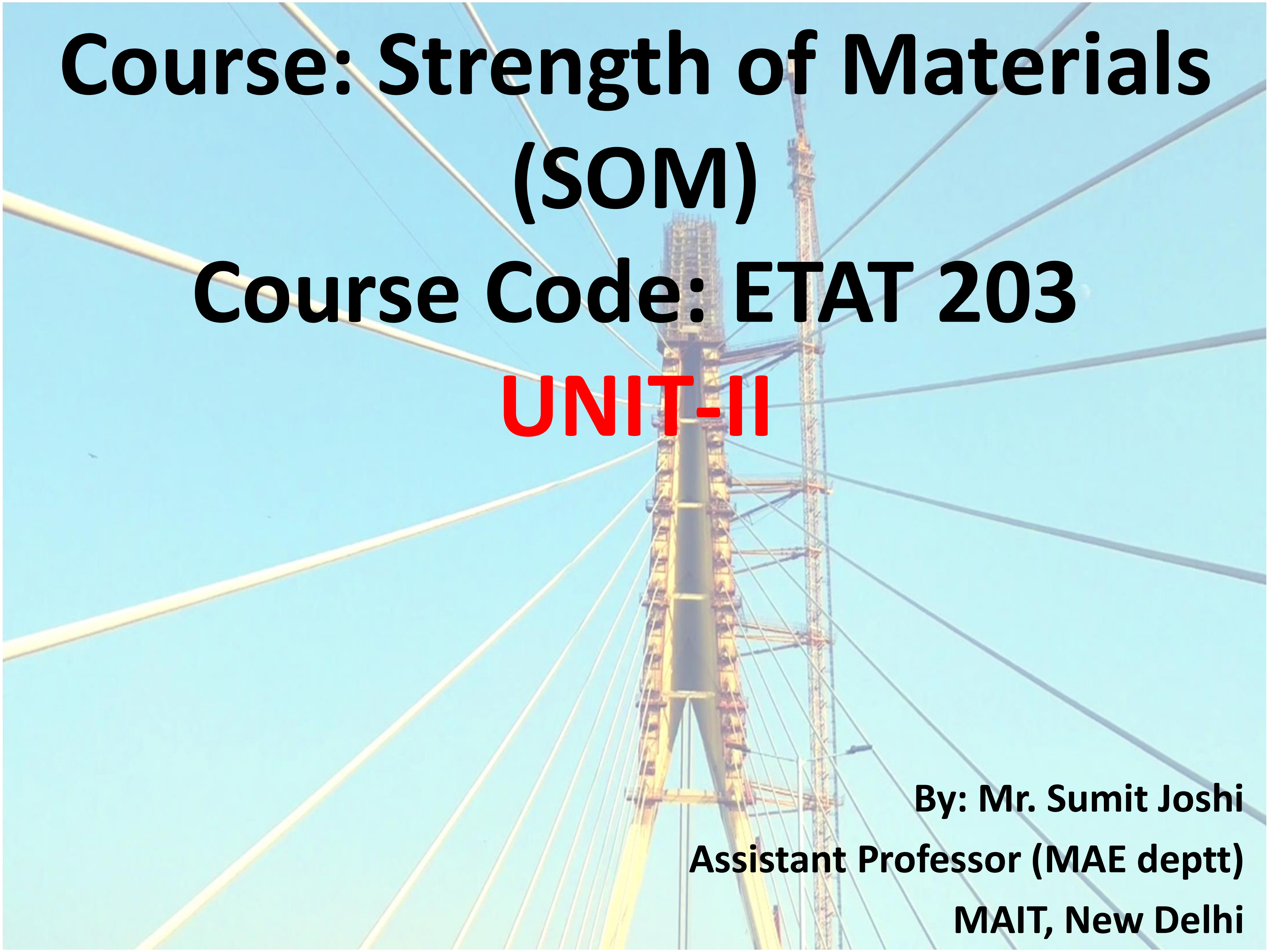

Course: Strength of Materials (SOM)

Course Code: ETAT 203

UNIT-II

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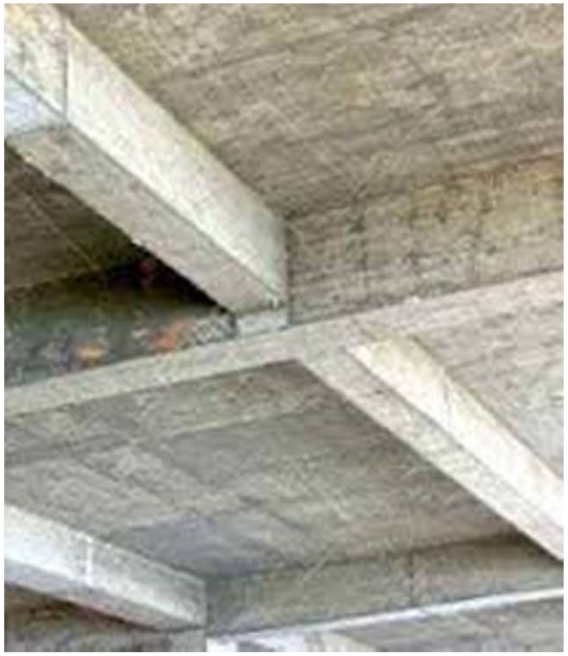
Beams

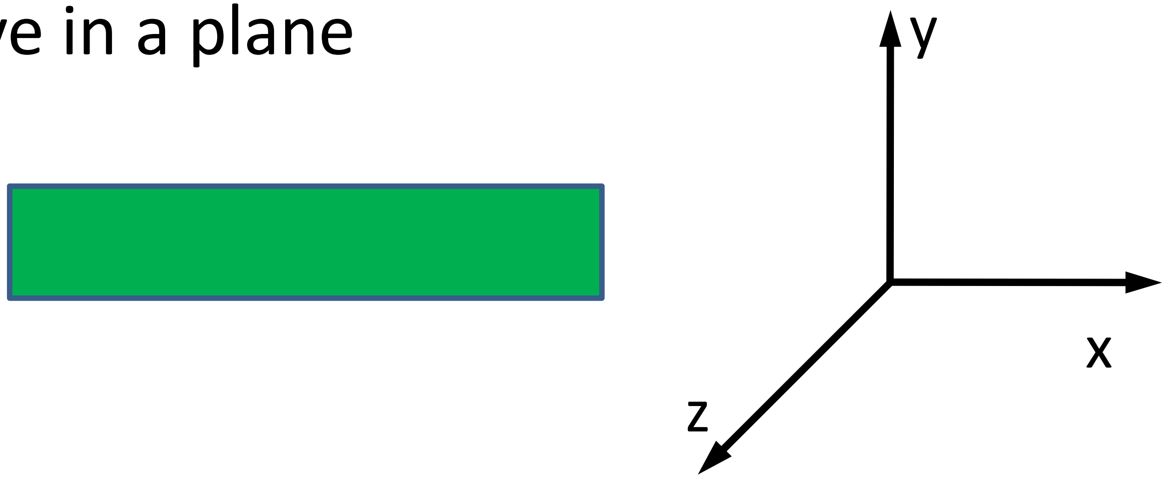
Beams are structural members used to carry loads usually perpendicular to their longitudinal axis. As the beams are used to carry the loads they are needed to be supported at different locations along their length.



Reactions and Supports

Reaction

If we consider the planar motion of a body then there are three different independent motions a body can have in a plane



1. Linear Motion along x axis.
2. Linear Motion along y axis.
3. Rotation about z axis.

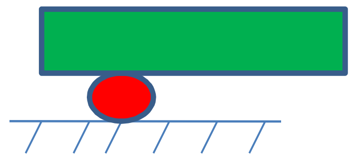
Reaction means restricting the motion. These restrictions are provided with the help of supports.

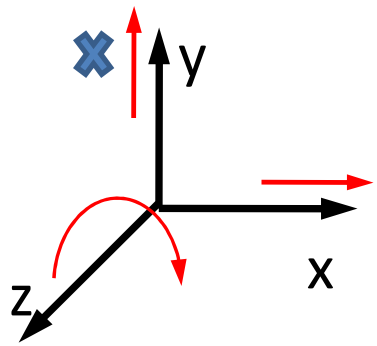
The independent movements a body can have is also called **DEGREES OF FREEDOM**

Types of Supports

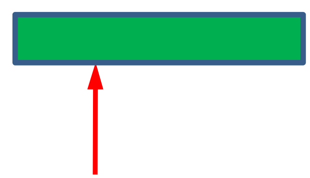
On the basis of the reaction offered by a support the supports are divided into three basic categories

1. Roller Support

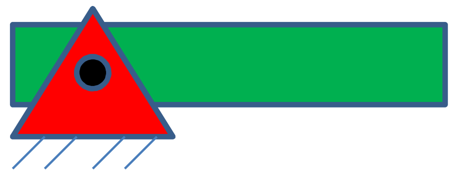


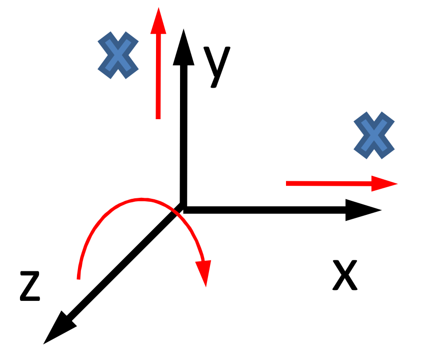


It means the roller support will provide a reaction in y direction only and free body diagram for the support will be

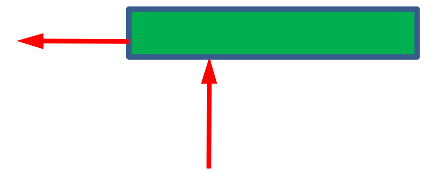


Hinged Support

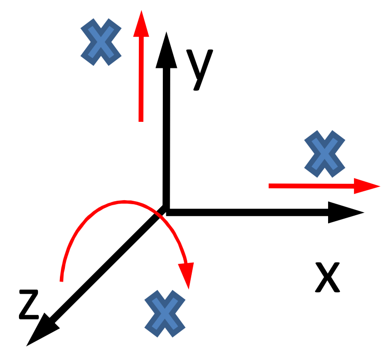




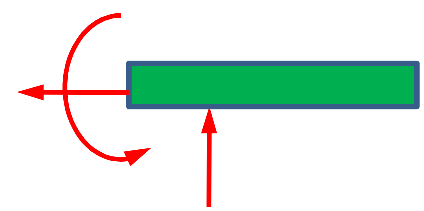
It means the hinged support will provide a reaction in y as well as in x directions only and free body diagram for the support will be



Fixed Support



It means the fixed support will provide a reaction in y as well as in x directions and restrict the rotation about z axis. Hence free body diagram for the support will be



Types of Beams

1. Cantilever Beam — Fixed at one end free at other

2. Cantilever Beam — Two Simple supports at two ends

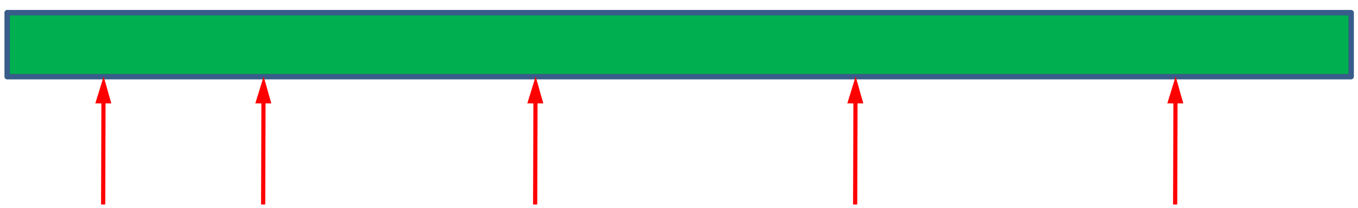
3. Overhanging Beam — Two Simple supports in between the length

4. Fixed Beam

Fixed at both the ends

5. Continuous Beam

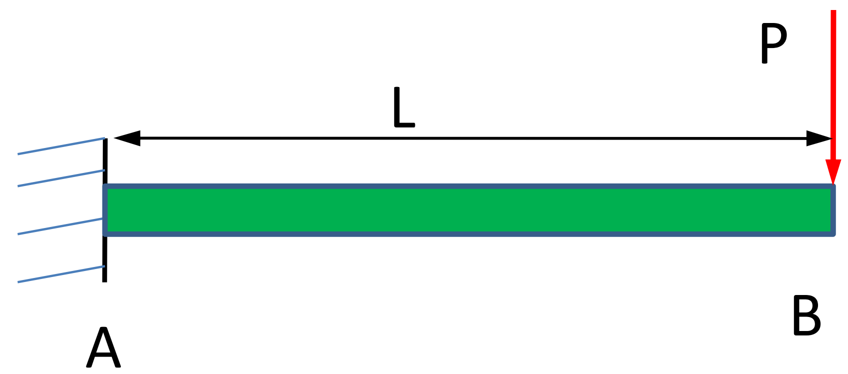
Have more than two supports



First three types of beams are statistically determinate beams whereas last two types of beams are statistically indeterminate.

Shear Force and Bending Moment

To find support reactions apply static equilibrium conditions



$$\sum F = 0$$

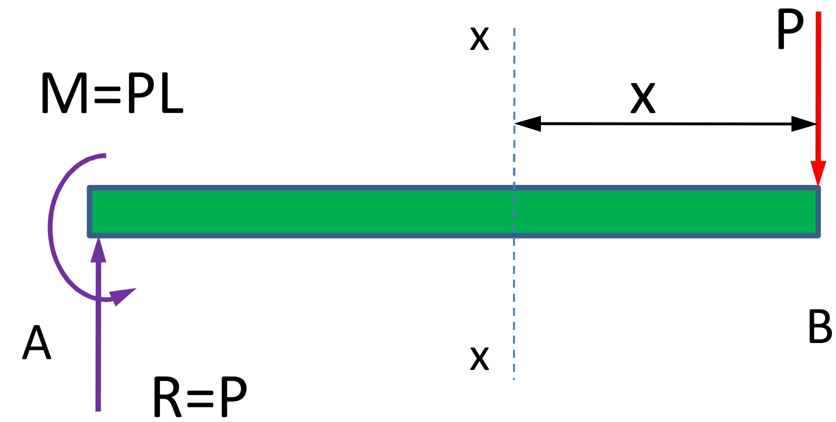
$$R - P = 0 \Rightarrow R = P$$

$$\sum M_A = 0$$

$$M - PL = 0 \Rightarrow M = PL$$

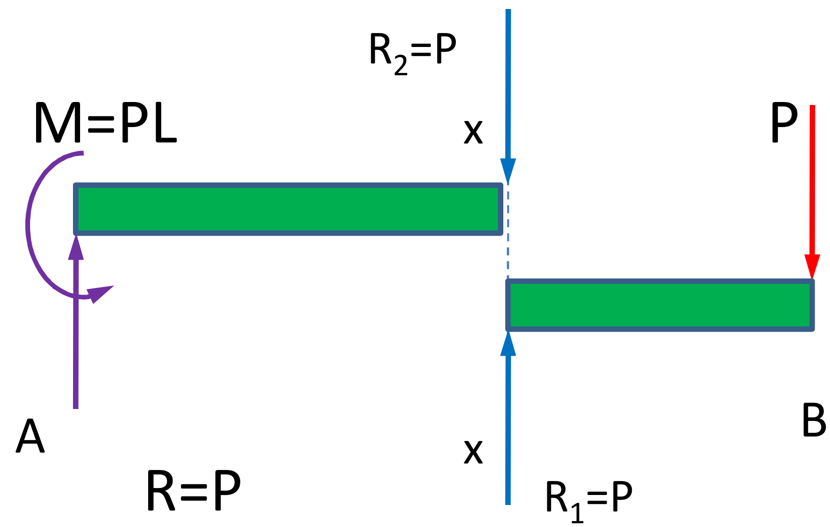
Now this beam can fail across section xx in two ways.

First way is that this beam is sheared along the section xx



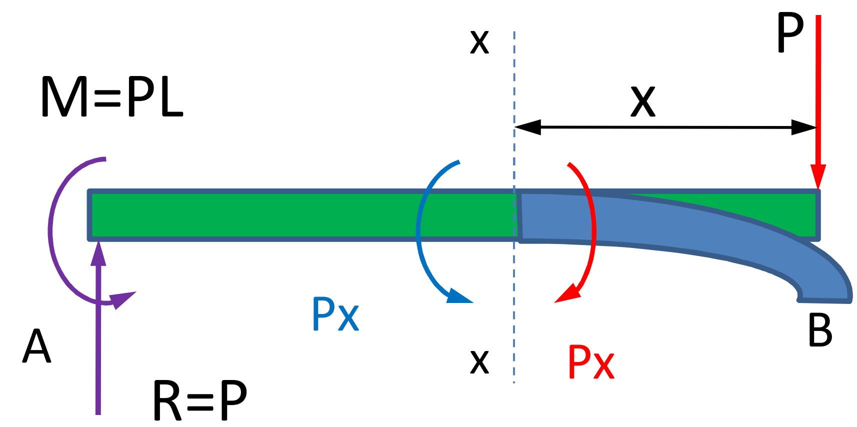
Now to avoid such failure if we consider RHS of beam, then there must be a reaction R_1 equal to P acting in upward direction at section xx

Similarly if we consider LHS, then there must be a reaction R_2 equal to P acting in downward direction at section xx



It means across section xx two parallel forces of equal magnitude are acting in opposite direction. These opposite parallel forces are known as Shear Force

Second way is that this beam starts bending along the section xx

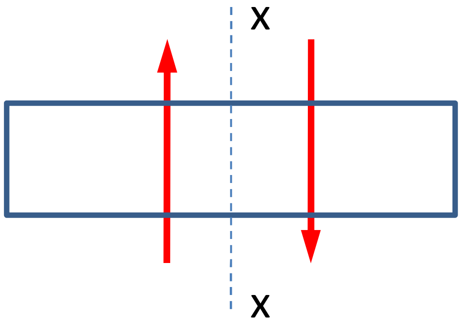


Net bending moment due to which beam starts bending across section xx is Px

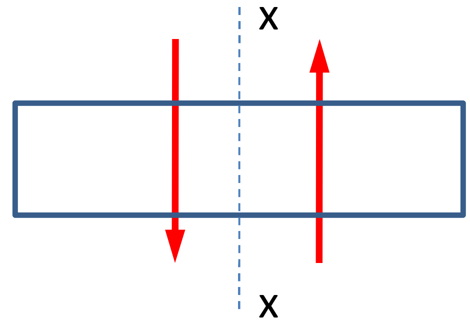
Now to avoid such failure if we consider RHS of beam, then there must be a reactive moment equal to Px acting in opposite direction is offered by section xx

It means across section xx two equal and opposite moments are acting. These equal and opposite moment across the section are known as Bending Moments.

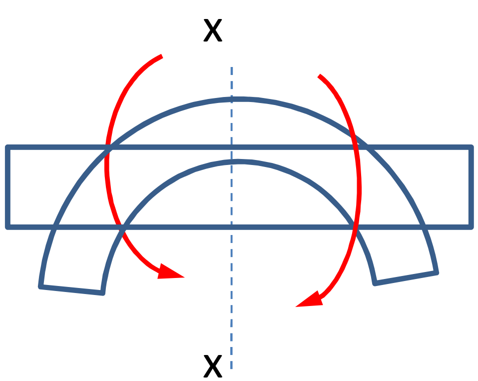
Sign Convention for Shear Force and Bending Moment



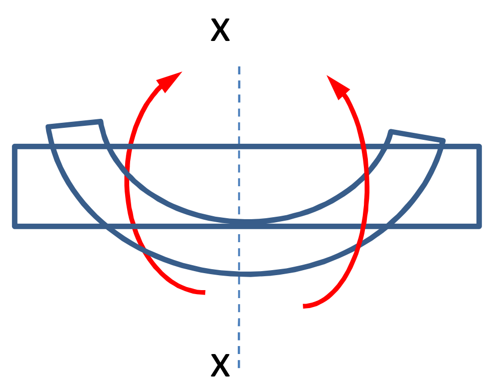
SF +ve



SF -ve



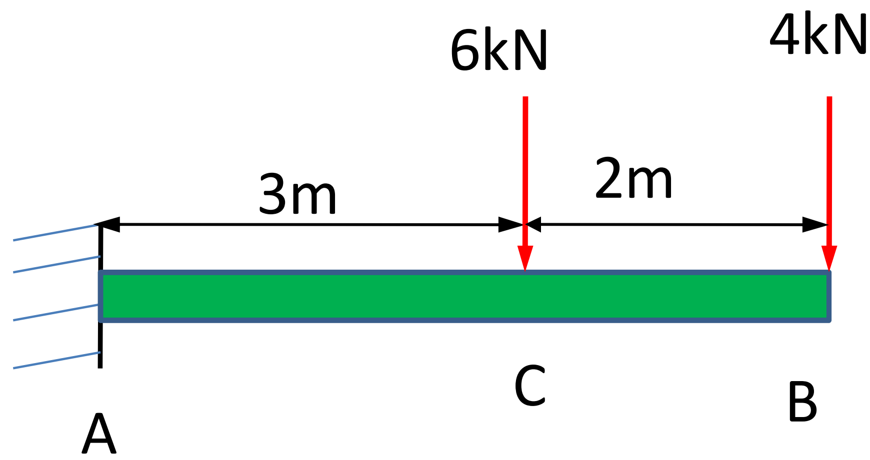
Hogging BM -ve



Sagging BM +ve

Step1

Draw the free body diagram for the beam



Step2

Calculate the reactions at the supports by applying static equilibrium conditions

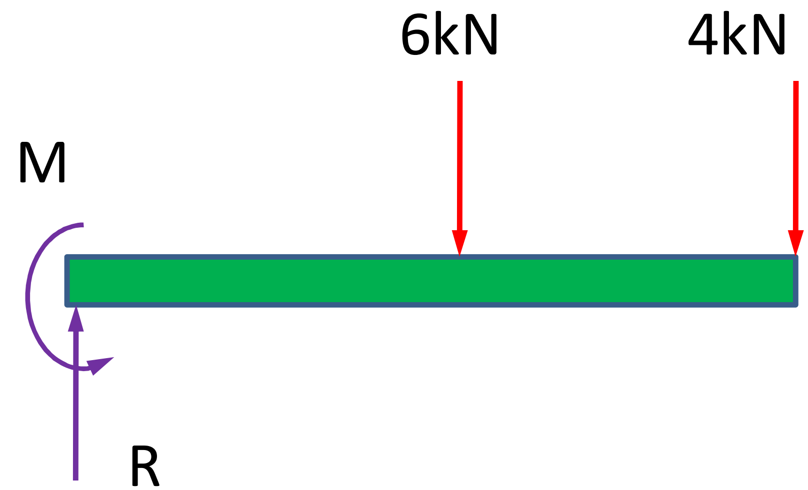
$$\sum F = 0$$

$$R - 6 - 4 = 0$$

$$R = 10\ kN$$

$$\sum M_A = 0$$

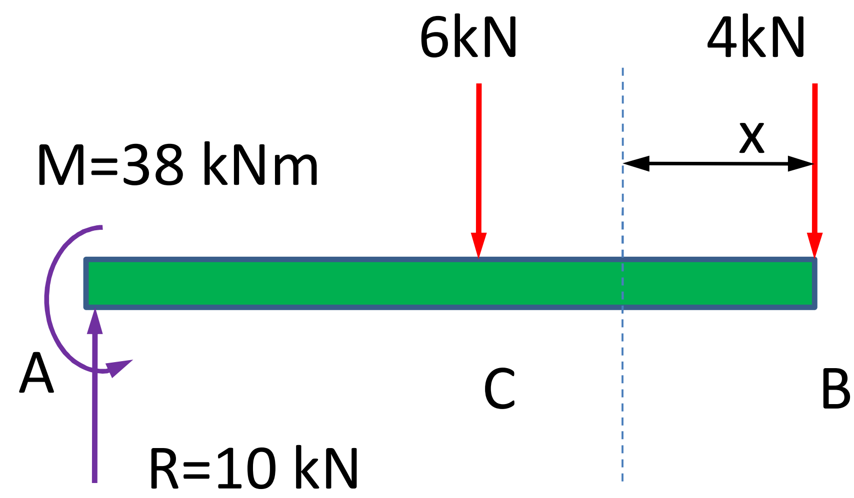
$$M - 6x3 - 4x5 = 0$$



$$M = 38\ kNm$$

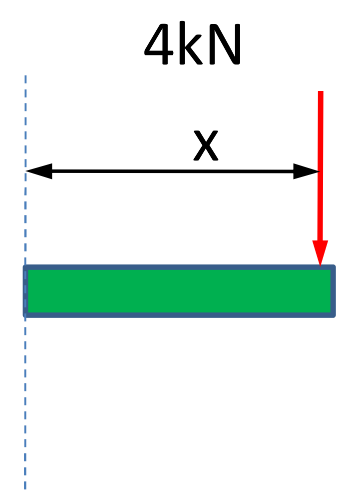
Step3

Consider the beam part wise.



For that let us take an arbitrary section xx at distance x from Point B between Part BC

$$Part\ BC\ (0 \leq x \geq 2m$$



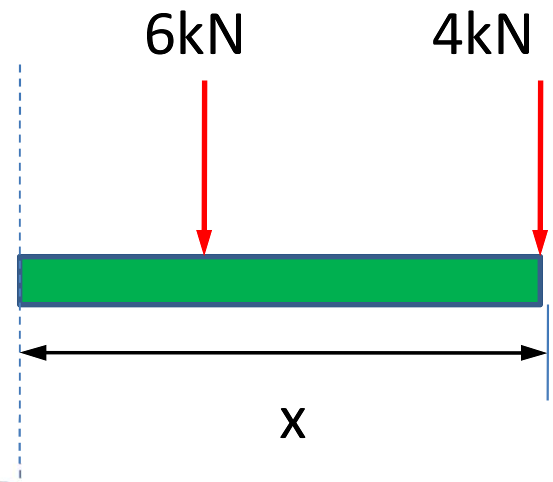
$$(SF)_{xx} = 4\ kN$$

$$(BM)_{xx} = -4x$$

$$(BM)_{B\ x=0} = -4\text{x}0 = 0\ \text{kNm}$$

$$(BM)_{C\ x=2m} = -4\text{x}2 = -8\ \text{kNm}$$

$$Part\ CA(2m \leq x \geq 5m)$$



$$(SF)_{xx} = 4 + 6 = 10\ kN$$

$$(BM)_{xx} = -4x - 6(x-2) = 10x + 12$$

$$(BM)_{C\ x=2m} = -10\text{x}2 + 12 = -8\ \text{kNm}$$

$$(BM)_{A\ x=5m} = -10\text{x}5 + 12 = -38\ \text{kNm}$$

$$Part\ BC\ (0 \le x \ge 2m$$

$$(SF)_{xx} = 4\ kN$$

$$(BM)_{xx} = -4x$$

$$(BM)_{B\ x=0} = -4\text{x}0 = 0\ \text{kNm}$$

$$(BM)_{C\ x=2m} = -4\text{x}2 = -8\ \text{kNm}$$

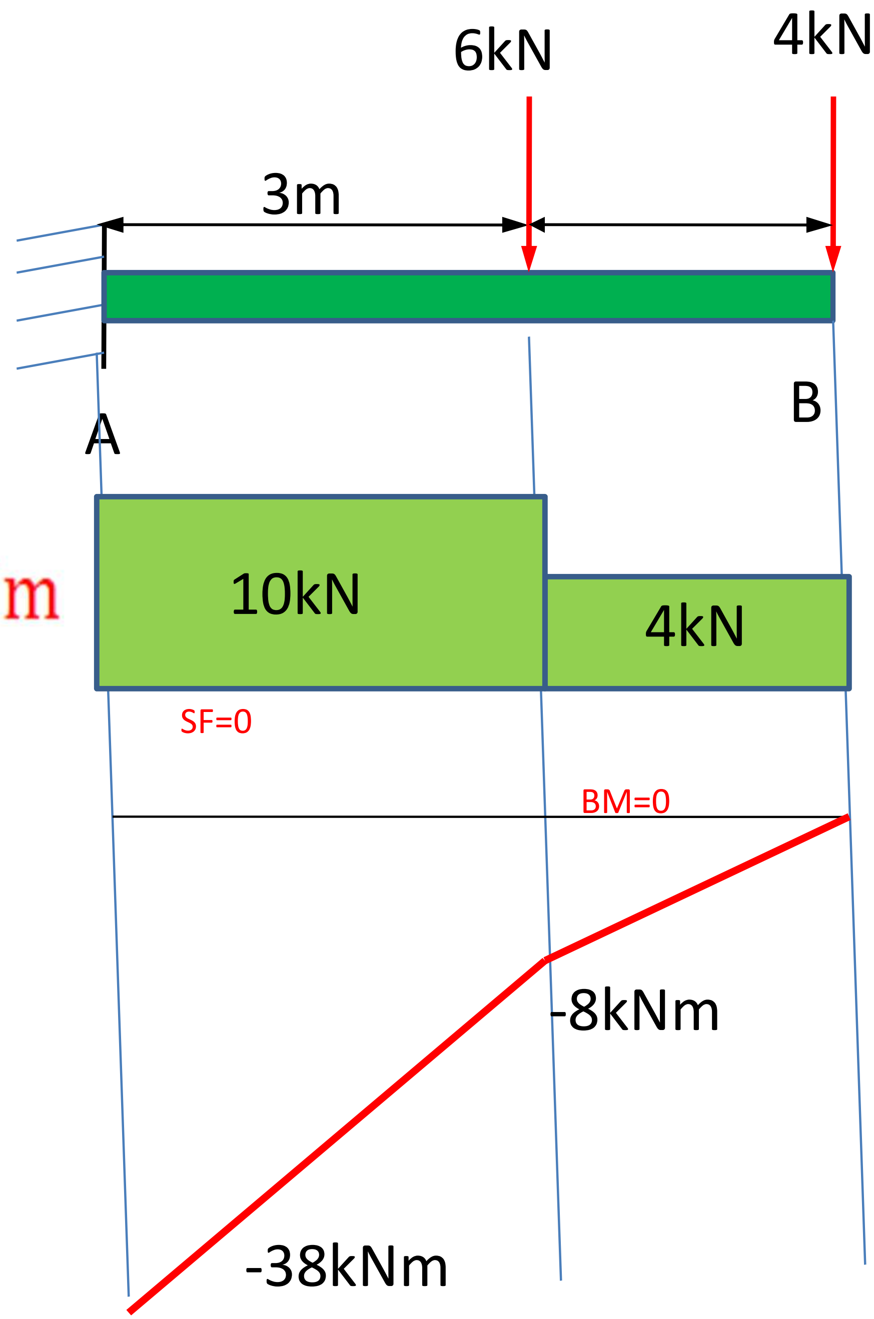
$$Part\ CA(2m \le x \ge 5m)$$

$$(SF)_{xx} = 10\ kN$$

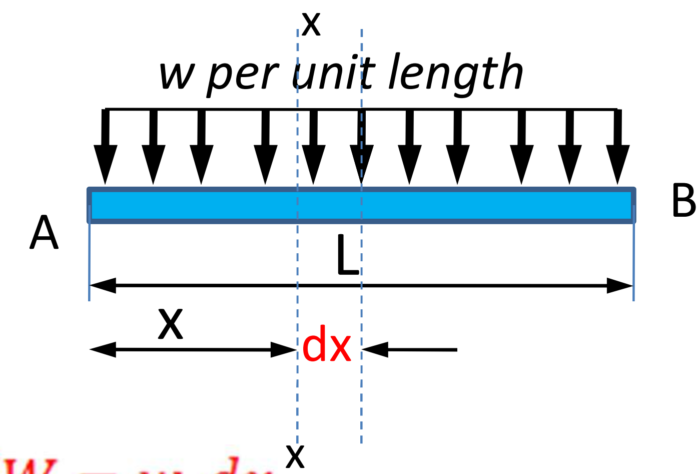
$$(BM)_{xx} = 10x + 12$$

$$(BM)_{C\ x=2m} = -8\ \text{kNm}$$

$$(BM)_{A\ x=5m} = -38\ \text{kNm}$$



Uniformly Distributed Load (UDL)



$$Net\ downward\ force\ on\ length\ dx = dW = w.dx$$

$$Total\ downward\ force\ on\ length\ L = W = \int_0^L w.dx = wL$$

Which is the area under load curve

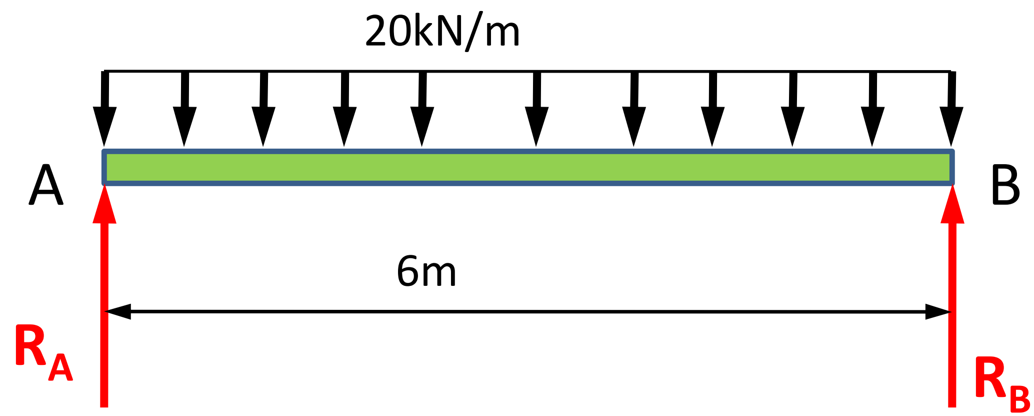
$$Moment\ due\ to\ force\ on\ dx\ about\ A = dM = dW.x = wx.dx$$

$$Total\ Moment\ due\ to\ about\ A = M = \int_0^L wx.dx = \frac{wL^2}{2} = wL\frac{L}{2}$$

Which is total load multiplied by centroidal distance of load curve from Point A

Step 1

To calculate reactions R_A and R_B apply conditions of static equilibrium.



$$\sum F = 0$$

$$R_A + R_B = 20x6 = 120\text{kN}$$

$$\sum M_A = 0$$

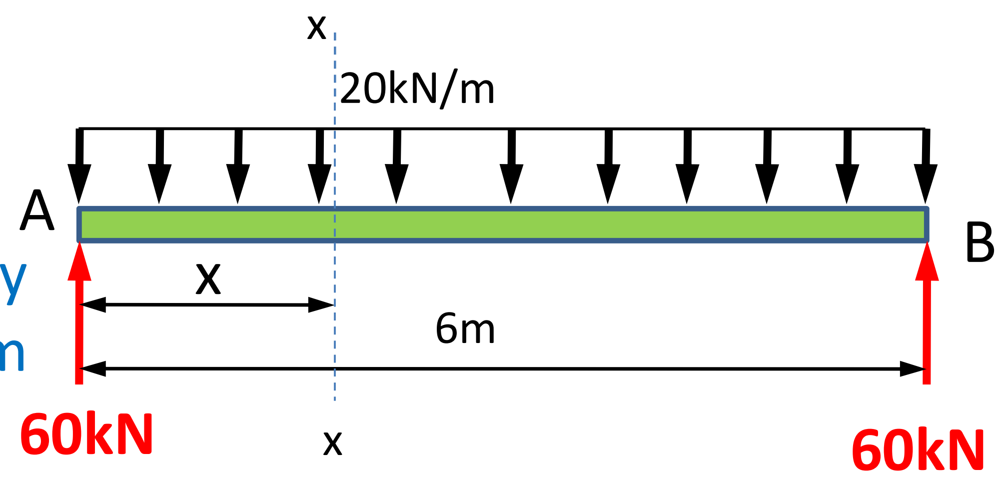
$$R_B x6 = 20x6x\frac{6}{2} = 360\text{kNm} \Rightarrow R_B = 60\text{kN}$$

From equation $R_A + R_B = 120\text{kN}$

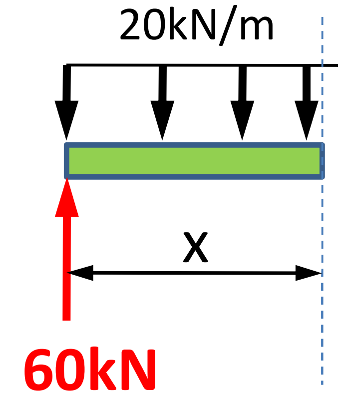
$$R_A = 60\text{kN}$$

Now considering the beam part wise

For that let us take an arbitrary section xx at distance x from Point A between Part AB



$$Part\ AB\ (0 \leq x \leq 6m)$$

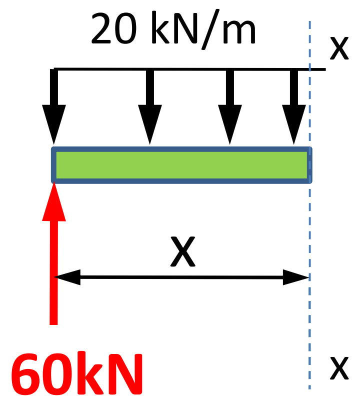


$$(SF)_{xx} = 60 - 20x$$

$$(SF)_{A\ x=0} = 60 - 20\text{x}0 = 60\text{kN}$$

$$(SF)_{B\ x=6m} = 60 - 20\text{x}6 = -60\text{kN}$$

$$(SF)_{xx} = 0 \quad \Rightarrow \quad 60 - 20x = 0\ or\ x = 3m$$



$$(BM)_{xx} = 60x - 20x\frac{x}{2} = 60x - 10x^2$$

$$(BM)_{A\ x=0} = 60\text{x}0 - 10\text{x}0^2 = 0kNm$$

$$(BM)_{B\ x=6m} = 60\text{x}6 - 10\text{x}6^2 = 0kNm$$

Now we calculate bending moment where Shear Force changes its sign

$$(BM)_{\ x=3m} = 60\text{x}3 - 10\text{x}3^2 = 90kNm$$

$$Part\ AB\ (0 \leq x \leq 6m)$$

$$(SF)_{A\ x=0m} = 60\text{kN}$$

$$(SF)_{B\ x=6m} = -60\text{kN}$$

Point where SF changes its sign

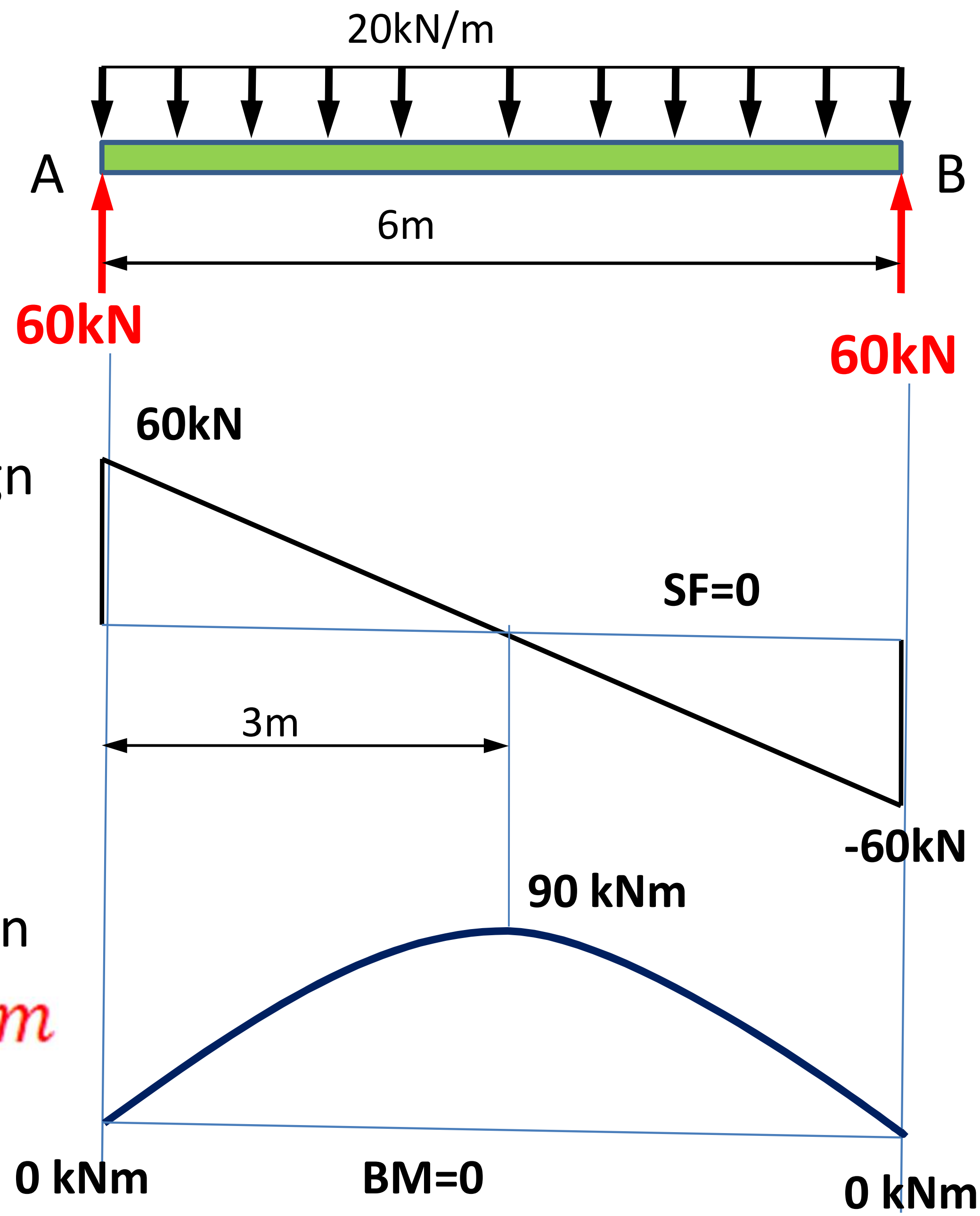
$$60 - 20x = 0\ or\ x = 3m$$

$$(BM)_{A\ x=0m} = 0kNm$$

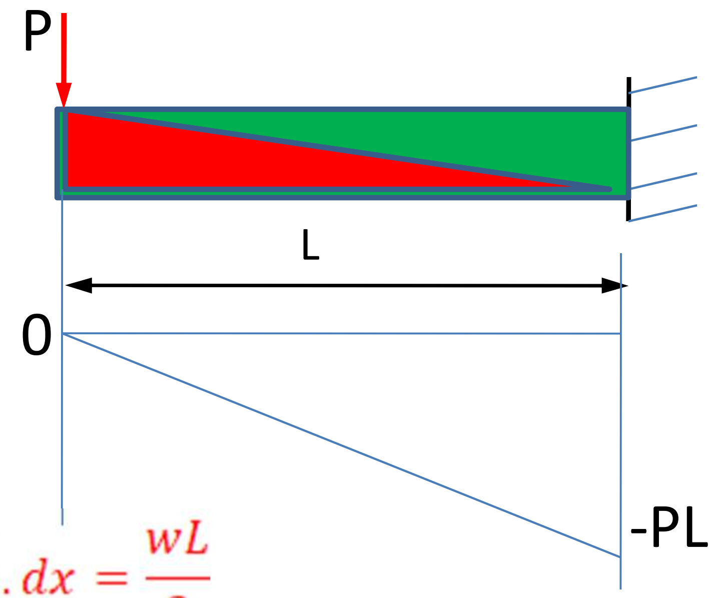
$$(BM)_{B\ x=6m} = 0kNm$$

BM where SF changed its sign

$$(BM)_{x=3m} = 90kNm$$



Uniformly Varying Load (UVL)



$$Rate\ of\ loading\ at\ distance\ x\ from\ A = \frac{wx}{L}$$

$$Net\ downward\ force\ on\ length\ dx = dW = \frac{wx}{L}.dx$$

$$Total\ downward\ force\ on\ length\ L = W = \int_0^L \frac{wx}{L}.dx = \frac{wL}{2}$$

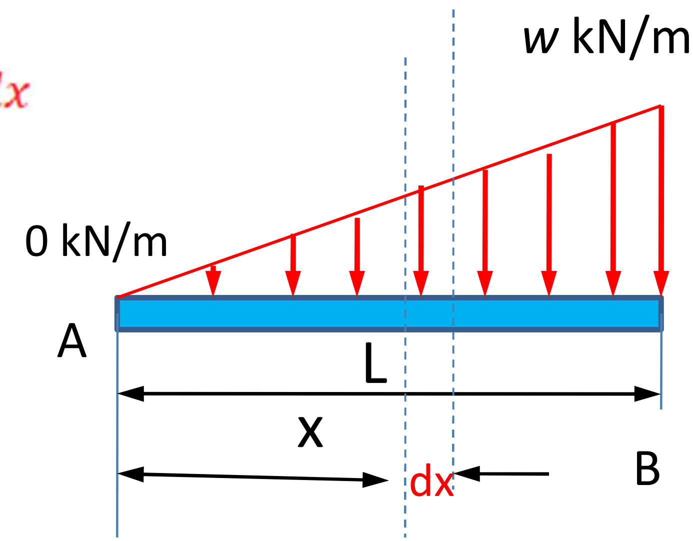
Which is the area under load curve

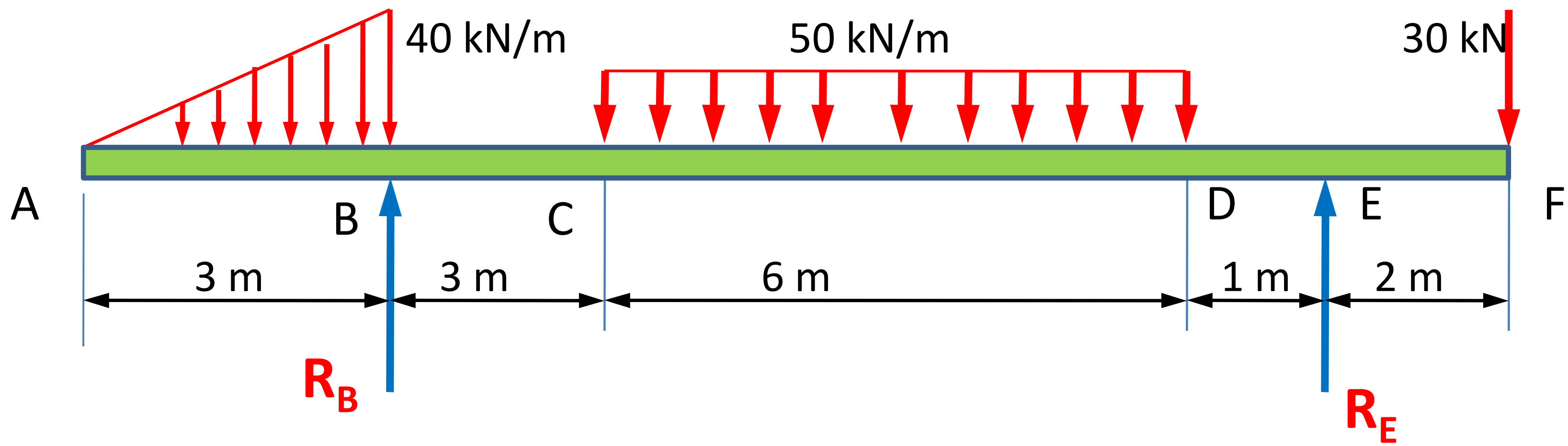
$$Moment\ due\ to\ force\ on\ dx\ about\ A = dM = \frac{wx}{L}x.dx$$

$$Total\ Moment\ due\ to\ about\ A = M = \int_0^L \frac{wx}{L}.xdx$$

$$= \frac{wL^2}{3} = \frac{wL}{2} \text{x} \frac{2L}{3}$$

Which is total load multiplied by centroidal distance of load curve from Point A





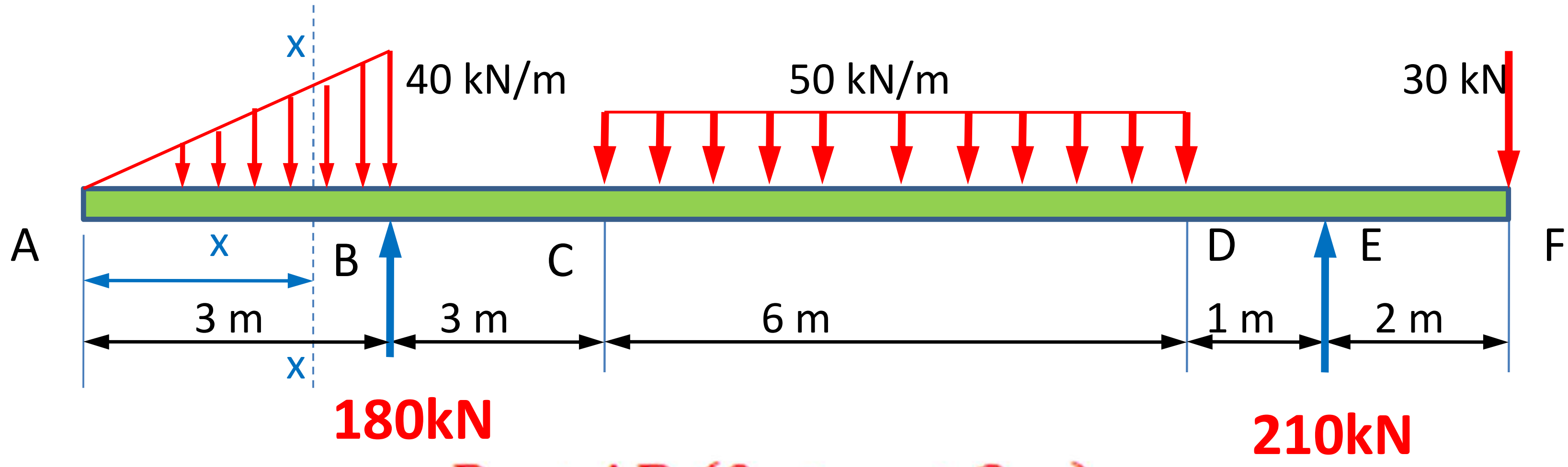
To calculate reactions at B and E, apply conditions of static equilibrium

$$\sum F = 0 \Rightarrow R_E + R_B = \frac{1}{2}\text{x3x40} + \text{6x50} + 30 = 390 \text{ kN}$$

$$\sum M_B = 0 \Rightarrow -30\text{xFB} + \text{R}_\text{E}\,\text{xEB} - 50\text{xCDx}\left(\text{CB} + \frac{\text{CD}}{2}\right) + \frac{1}{2}\text{x40xABx}\left(\frac{\text{AB}}{3}\right) = 0$$

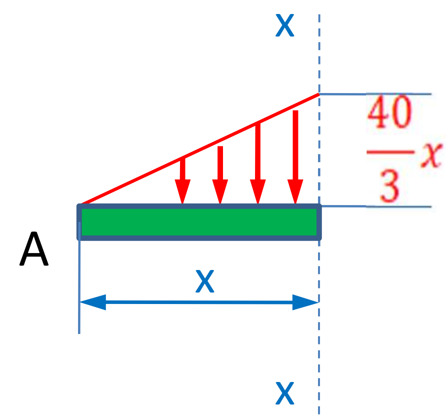
$$-30\text{x12} + \text{R}_\text{E}\,\text{x10} - \text{50x6x6} + \frac{1}{2}\text{x40x3x1} = 0 \Rightarrow \text{R}_\text{E} = 210 \text{ kN}$$

$$\text{R}_\text{B} = 180 \text{ kN}$$



Part AB $(0 \le x \le 3m)$

$$(SF)_{xx} = -\frac{1}{2} \times \frac{40}{3} x \times x = -\frac{20}{3} x^2$$



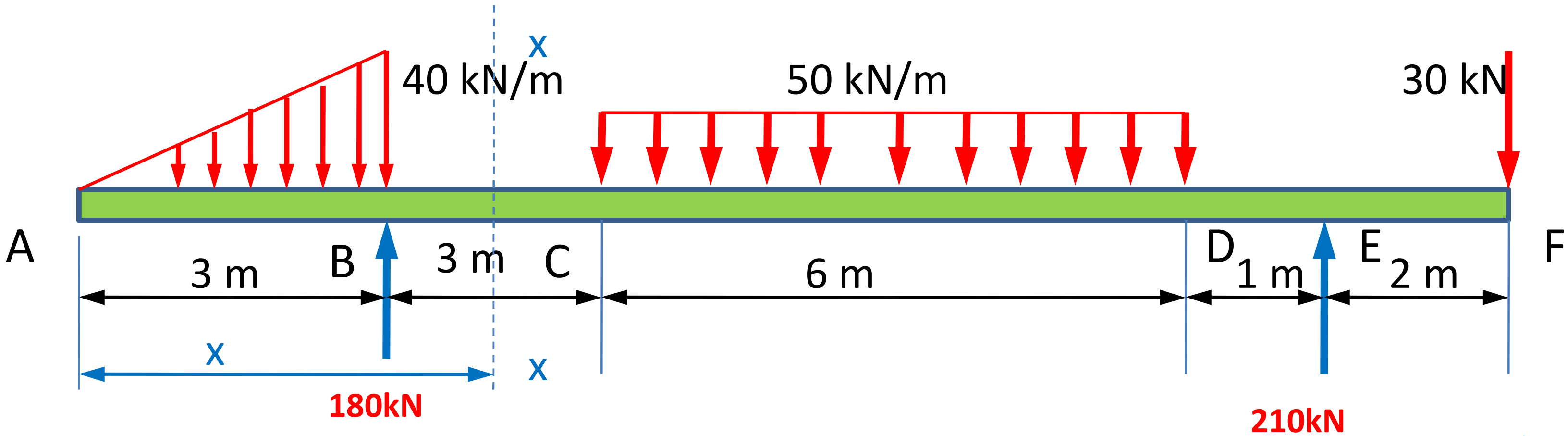
$$(SF)_{A\ x=0} = -\frac{20}{3} \times 0^2 = 0kN$$

$$(SF)_{B\ x=3m} = -\frac{20}{3} \times 3^2 = -60kN$$

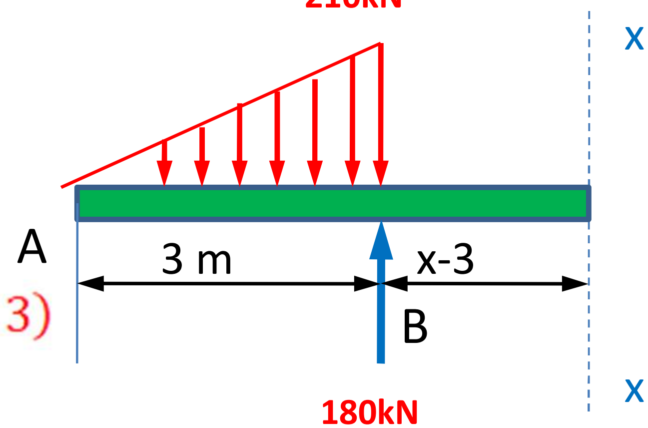
$$(BM)_{xx} = -\frac{1}{2} \times \frac{40}{3} x \times x \times \frac{x}{3} = -\frac{20}{9} x^3$$

$$(BM)_{A\ x=0} = -\frac{20}{9} \times 0^2 = 0kNm$$

$$(BM)_{B\ x=3m} = -\frac{20}{9} \times 3^3 = -60kNm$$



$Part\ BC\ (3m \leq x \leq 6m)$



$$(SF)_{xx} = -\frac{1}{2} \times 40 \times 3 + 180 = 120kN$$

$$(BM)_{xx} = -\frac{1}{2} \times 40 \times 3 \times \left(x - 3 + \frac{3}{3}\right) + 180(x - 3)$$

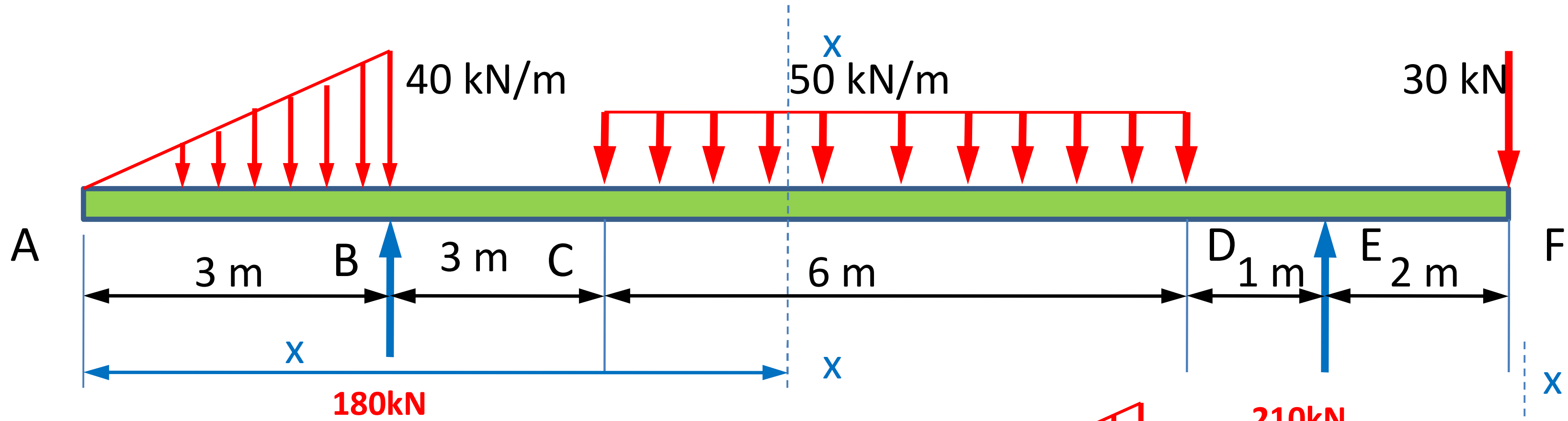
$$= 120x - 420$$

$$(BM)_{B\ x=3m} = 120 \times 3 - 420 = -60kNm$$

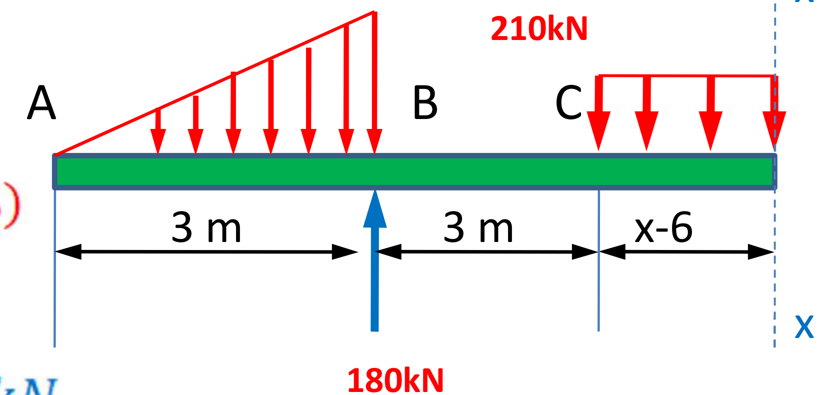
$$(BM)_{C\ x=6m} = 120 \times 6 - 420 = 300kNm$$

To find Point of Contraflexure or Point of Inflexion

$$(BM)_{xx} = 120x - 420 = 0 \quad \Rightarrow \quad x = 3.5\ m$$



Part CD ($6m \le x \le 12m$)



$$(SF)_{xx} = -\frac{1}{2} \times 40 \times 3 + 180 - 50(x-6)$$

$$= -50x + 420$$

$$(SF)_{C\ x=6m} = -50 \times 6 + 420 = 120kN$$

$$(SF)_{D\ x=12m} = -50 \times 12 + 420 = -180kN$$

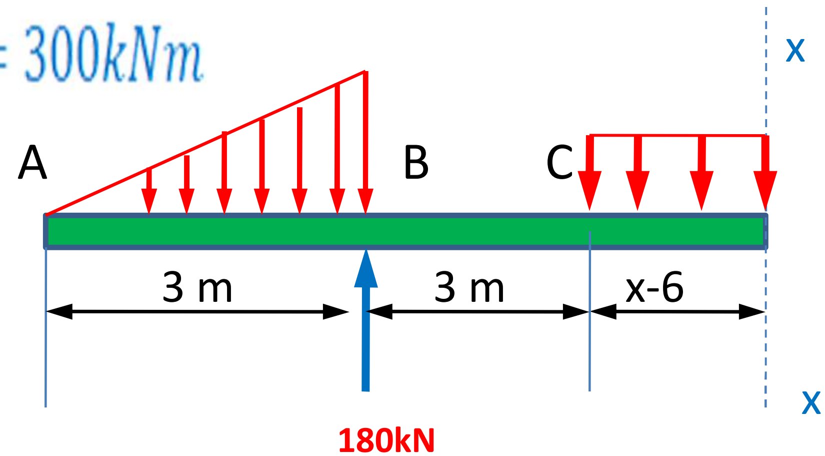
To find Point where SF changes its sign

$$(SF)_{xx} = -50x + 420 = 0 \quad \Longrightarrow \quad x = 8.4\ m$$

$$(BM)_{xx} = -\frac{1}{2} \times 40 \times 3 \times \left(x - 3 + \frac{3}{3}\right) + 180(x-3) - 50(x-6)\frac{(x-6)}{2}$$

$$= -25x^2 + 420x - 1320$$

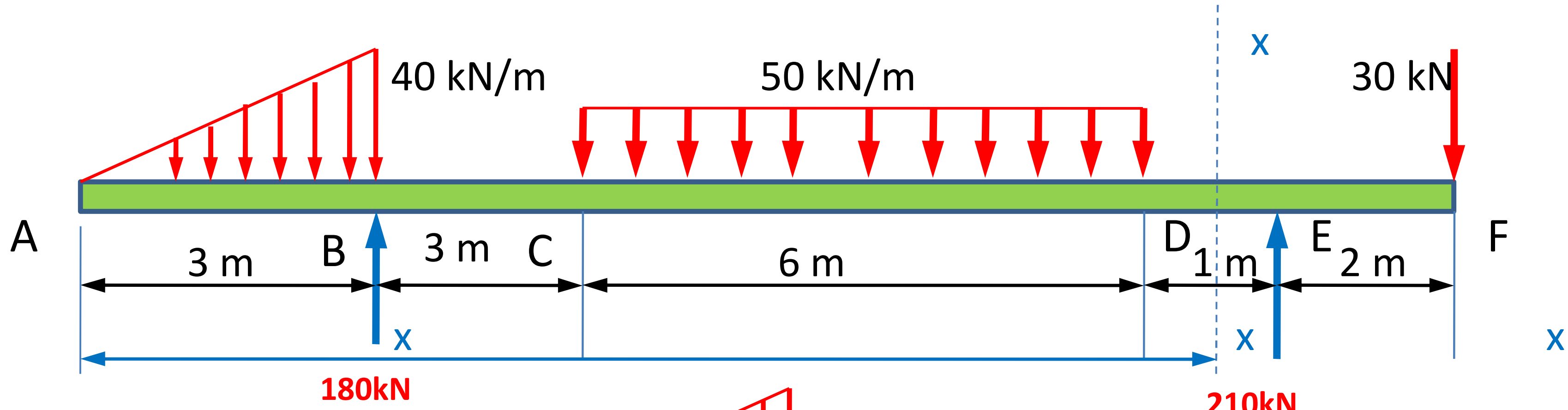
$$(BM)_{C\ x=6m} = -25 \times 6^2 + 420 \times 6 - 1320 = 300kNm$$

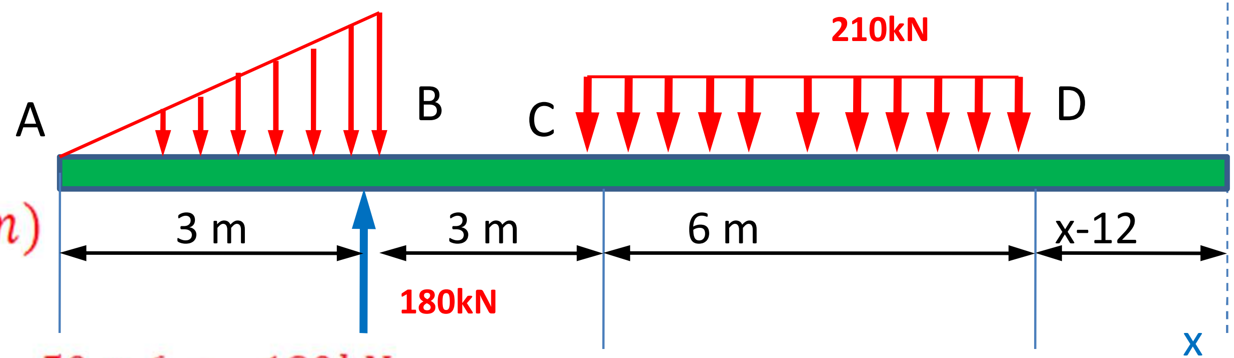


$$(BM)_{D\ x=12m} = -25 \times 12^2 + 420 \times 12 - 1320 = 120kNm$$

To find Maximum BM at the point where SF changes its sign

$$(BM)_{\ x=8.4m} = -25 \times 8.4^2 + 420 \times 8.4 - 1320 = 444kNm$$





Part DE $(12m \le x \le 13m)$

$$(SF)_{xx} = -\frac{1}{2}\times 40\times 3 + 180 - 50\times 6 = -180kN$$

$$(BM)_{xx} = -\frac{1}{2}\times 40\times 3\times\left(x-3+\frac{3}{3}\right)+180(x-3)-50\times 6\times\left(x-12+\frac{6}{2}\right)$$

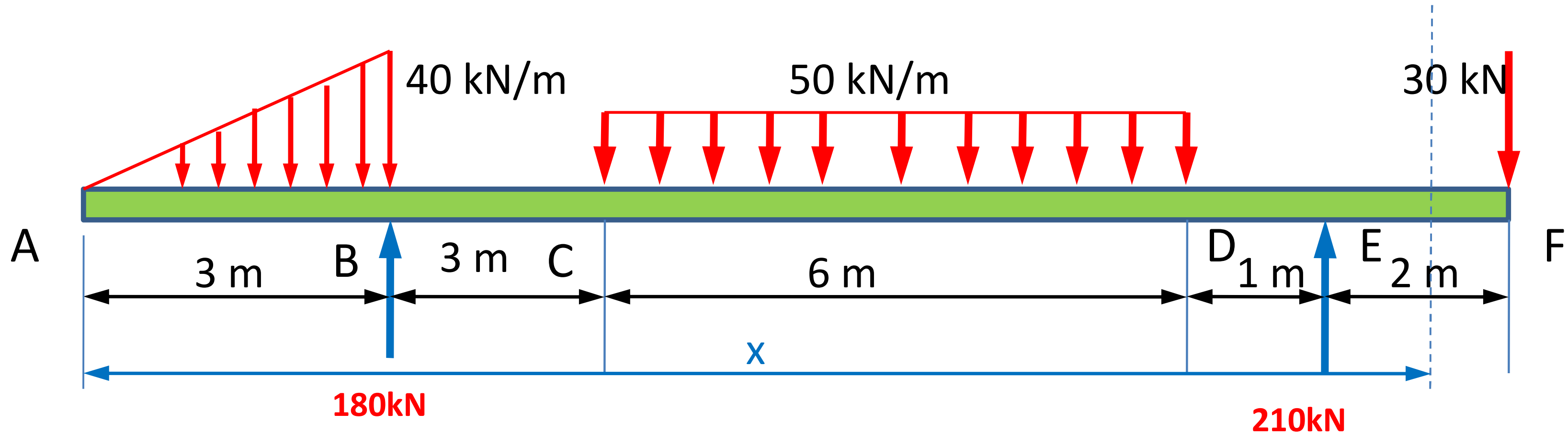
$$= -180x + 2280$$

$$(BM)_{D\ x=12m} = -180\times 12 + 2280 = 120kNm$$

$$(BM)_{E\ x=13m} = -180\times 13 + 2280 = -60kNm$$

To find Point of Contraflexure or Point of Inflexion

$$(BM)_{xx} = 180x - 2280 = 0 \quad \Rightarrow \quad x = 12.67\ m$$



$Part\ EF\ (13m \leq x \leq 15m)$

$$(SF)_{xx} = -\frac{1}{2} \times 40 \times 3 + 180 - 50 \times 6 + 210 = 30kN$$

$$(BM)_{xx} = -\frac{1}{2} \times 40 \times 3 \times (x-2) + 180(x-3) - 50 \times 6 \times (x-9) + 210(x-13)$$

$$= 30x - 450$$

$$(BM)_{E\ x=13m} = 30 \times 13 - 450 = -60kNm$$

$$(BM)_{F\ x=15m} = 30 \times 15 - 450 = 0kNm$$

Part AB $(0 \leq x \leq 3m)$

$$(SF)_{xx} = -\frac{20}{3}x^2$$

$$(SF)_{A\ x=0} = 0kN$$

$$(SF)_{B\ x=3m} - 60kN$$

$$(BM)_{xx} = -\frac{20}{9}x^3$$

$$(BM)_{A\ x=0} = 0kNm$$

$$(BM)_{B\ x=3m} = -60kNm$$

Part BC $(3m \leq x \leq 6m)$

$$(SF)_{xx} = 120kN$$

$$(BM)_{xx} = 120x - 420$$

$$(BM)_{B\ x=3m} = -60kNm$$

$$(BM)_{C\ x=6m} = 300kNm$$

Point of contraflexure is at

$$x = 3.5\ m$$

Part CD $(6m \leq x \leq 12m)$

$$(SF)_{xx} = -50x + 420$$

$$(SF)_{C\ x=6m} = 120kN$$

$$(SF)_{D\ x=12m} = -180kN$$

Point where SF changes its sign

$$x = 8.4\ m$$

$$(BM)_{xx} = -25x^2 + 420x - 1320$$

$$(BM)_{C\ x=6m} = 300kNm$$

$$(BM)_{D\ x=12m} = 120kNm$$

$$(BM)_{\ x=8.4m} = 444kNm$$

Part DE $(12m \leq x \leq 13m)$

$$(SF)_{xx} = -180kN$$

$$(BM)_{xx} = -180x + 2280$$

$$(BM)_{D\ x=12m} = 120kNm$$

$$(BM)_{E\ x=13m} = -60kNm$$

Point of contraflexure is at

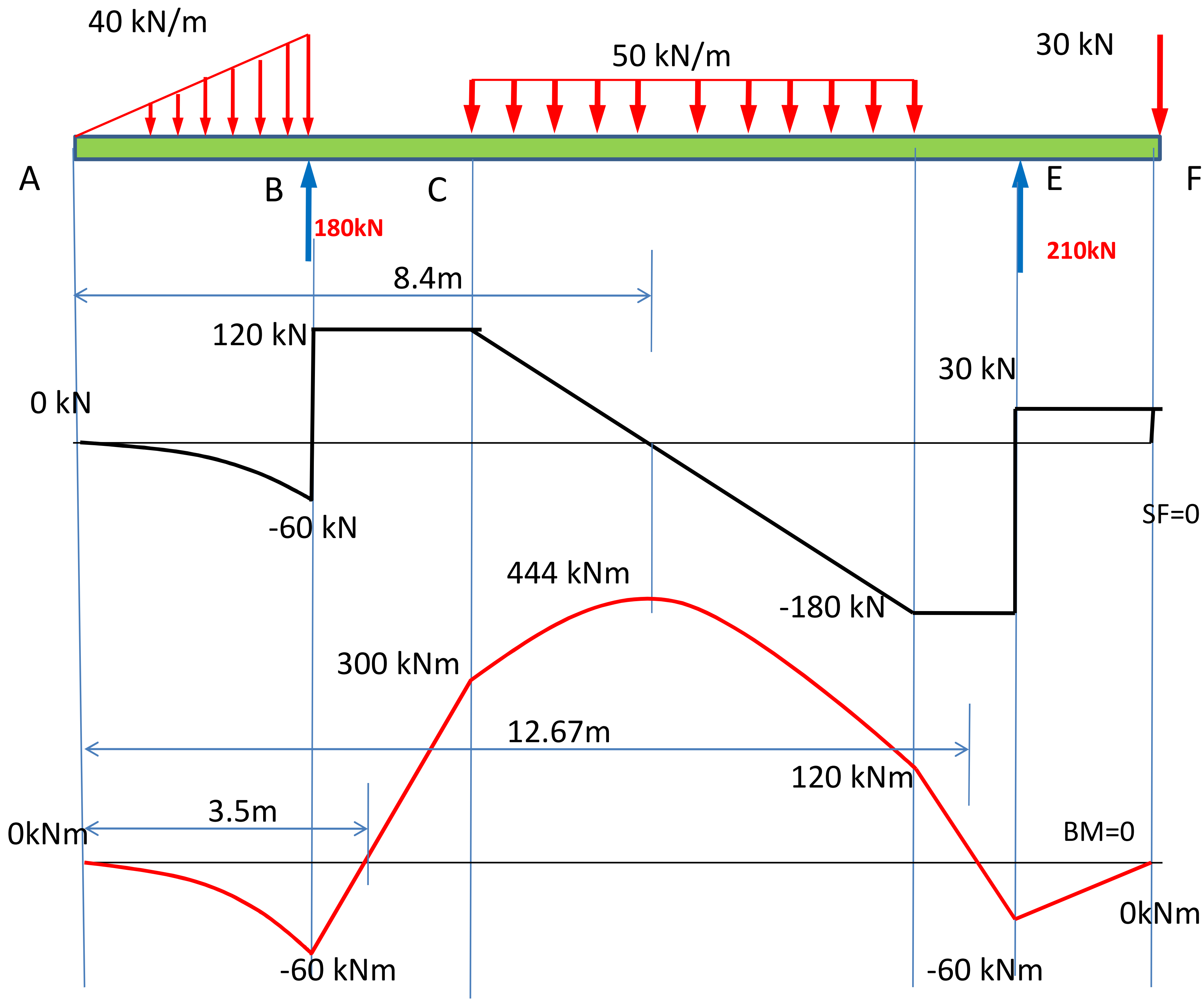
$$x = 12.67\ m$$

Part EF $(13m \leq x \leq 15m)$

$$(SF)_{xx} = 30kN$$

$$(BM)_{E\ x=13m} = -60kNm$$

$$(BM)_{F\ x=15m} = 0kNm$$



40 kN/m
50 kN/m
30 kN
A
B
C
E
F
180kN
210kN
8.4m
120 kN
0 kN
30 kN
-60 kN
SF=0
444 kNm
-180 kN
300 kNm
12.67m
120 kNm
3.5m
0kNm
BM=0
0kNm
-60 kNm
-60 kNm

Points to Remember

- Rate of change of SF at any point gives the rate of loading at that point

$$\frac{d}{dx}(SF)_{xx} = Rate\ of\ loading$$

- Rate of change of BM at any point gives the value of shear force at that point

$$\frac{d}{dx}(BM)_{xx} = (SF)_{xx}$$

- Point at which SF changes its sign, there is local maxima for BM

- Point at which BM changes its sign is called **Point of Contraflexure or Point of Inflexion**

Points to Remember

- Vertical reaction at any support is equal to height of SFD at that support and vice versa

- At fixed supports, both S.F. and B.M. are non zero

- At simple supports, S.F. is non zero and B.M. is zero

- In presence of concentrated point loads, sudden change (sudden jump) in shear force takes place i.e., sudden jump is equal to magnitude of concentrated load acting at that point.

- Between two concentrated point loads SF is constant i.e. SFD consists of horizontal line

Points to Remember

LOAD	S.F. VARIATION	B.M. VARIATION
ZERO (between two point load)	CONSTANT (Horizontal Line)	LINEAR (Inclined line)
CONSTANT (UDL)	LINEAR (Inclined line)	PARABOLIC
VARYING (UVL)	PARABOLIC	CUBIC PARABOLIC
Between two concentrated moments	ZERO	CONSTANT (Horizontal Line)