Course: Machine Design-I: MEC-212

Unit-2 Design of Shafts

UNIT-II

Shafts, keys and couplings: Transmission Shafts, materials, design of shafts on strength & rigidity basis and under combined torsional and bending loads as per ASME code. Keys, types and applications. Design of rigid and pin bushed flexible couplings.

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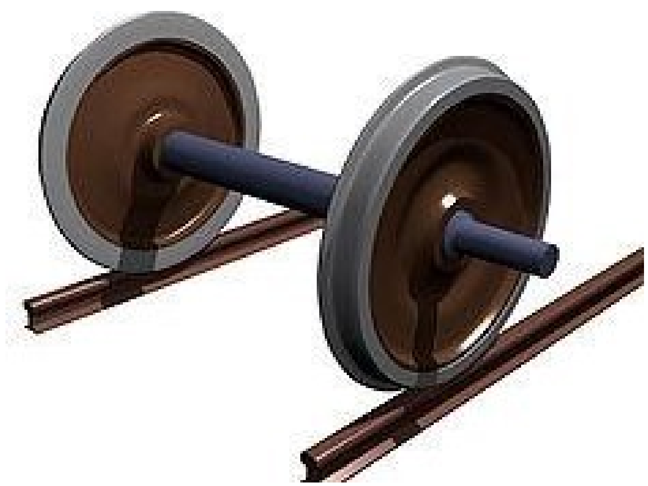
Introduction

- In machinery, the general term "shaft" refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc., and which is subjected to torsion and to transverse or axial loads acting singly or in combination.



Types of Shafts

1 Machine "shaft" e g. Crank shaft of IC engine

2 Transmission shaft e g. shaft – motor to compressor

- Solid
- Hollow
- Stepped

3Axle e g. Real axle of railway wagon, axle of car

4] Spindle e g. Spindle of drilling m/c

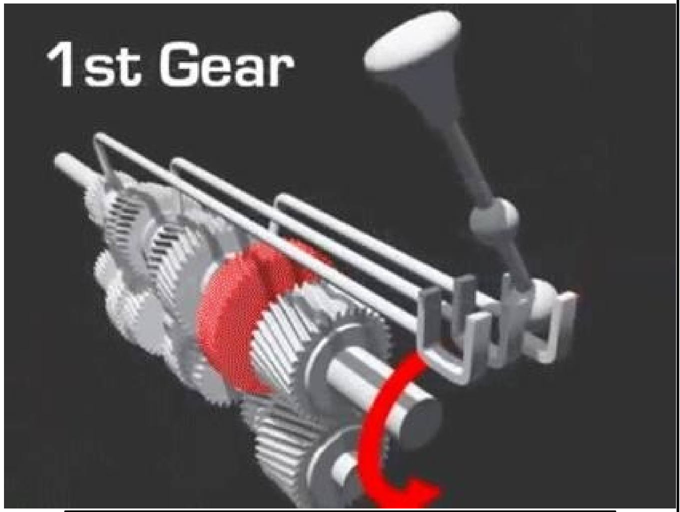
5] Countershaft e g. Secondary shaft

Transmission Shafts

- Rotating member usually of circular cross section used to transmit power or motion.
- Supports transmission elements like gears, pulleys and sprockets.
- A transmission shaft supporting a gear in a speed reducer is shown in Fig.
- The shaft is always stepped with maximum diameter in the middle.
- The portion of minimum diameter at the two ends, where bearings are mounted.

Transmission Shafts

- The transmission shafts are made of medium carbon steels with a carbon content from 0.15 to 0.40 per cent such as 30C8 or 40C8.
- Commercial shafts are made of low carbon steels.
- It produced by hot-rolling and finished to size either by cold-drawing or by turning and grinding.
- Steel bars up to 200 mm in diameter are commercially available.



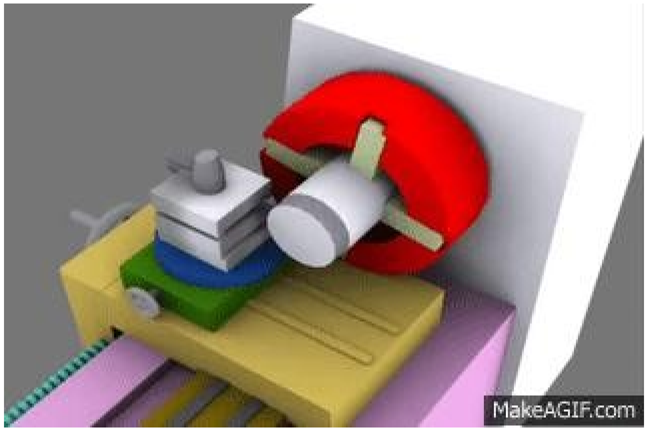
Diameters (mm)			
5	20	45	90
6	22	50	100
8	25	55	110
10	28	60	120
12	30	65	140
14	32	70	160
16	35	75	180
18	40	80	200

Axel

- An “axle” is a non-rotating member that supports wheels, pulleys,… and carries no torque.
- Axle-supports rotating elements like wheels, hoisting drums. Used in rear axle of a railway wagon, automobile rear axle.

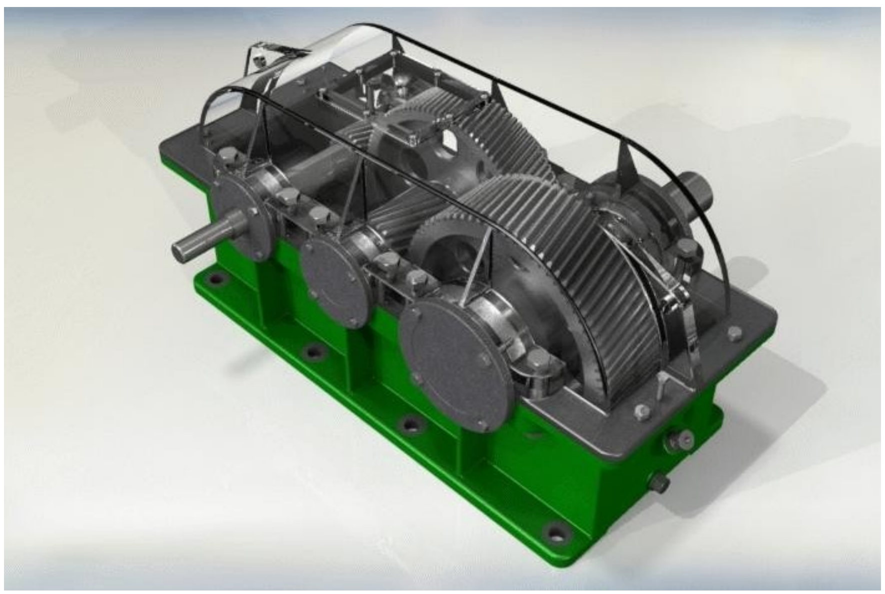
Spindle

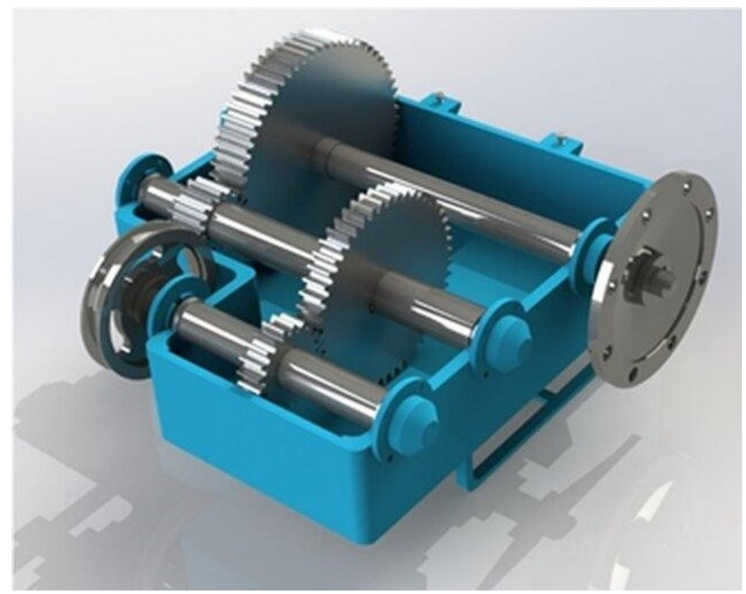
- A “spindle” is a short shaft. Terms such as line shaft, head shaft, stub shaft, transmission shaft, countershaft, and flexible shaft are names associated with special usage.
- A spindle is a short rotating shaft , used in all machine tools such as the small drive shaft of a lathe or the spindle of a drilling machine.



Countershaft

It is a secondary shaft, driven by the main shaft and from which the power is supplied to a machine component. rotates 'counter' to the direction of the main shaft. It is used in multi-stage gearboxes.





Stresses Induced in the Shaft

Transmission shafts are subjected to axial tensile force, bending moment or torsional moment or their combinations.

Tensile stress

$$\sigma_t = \frac{P}{\left[\frac{\pi d^2}{4}\right]}$$

Bending stress

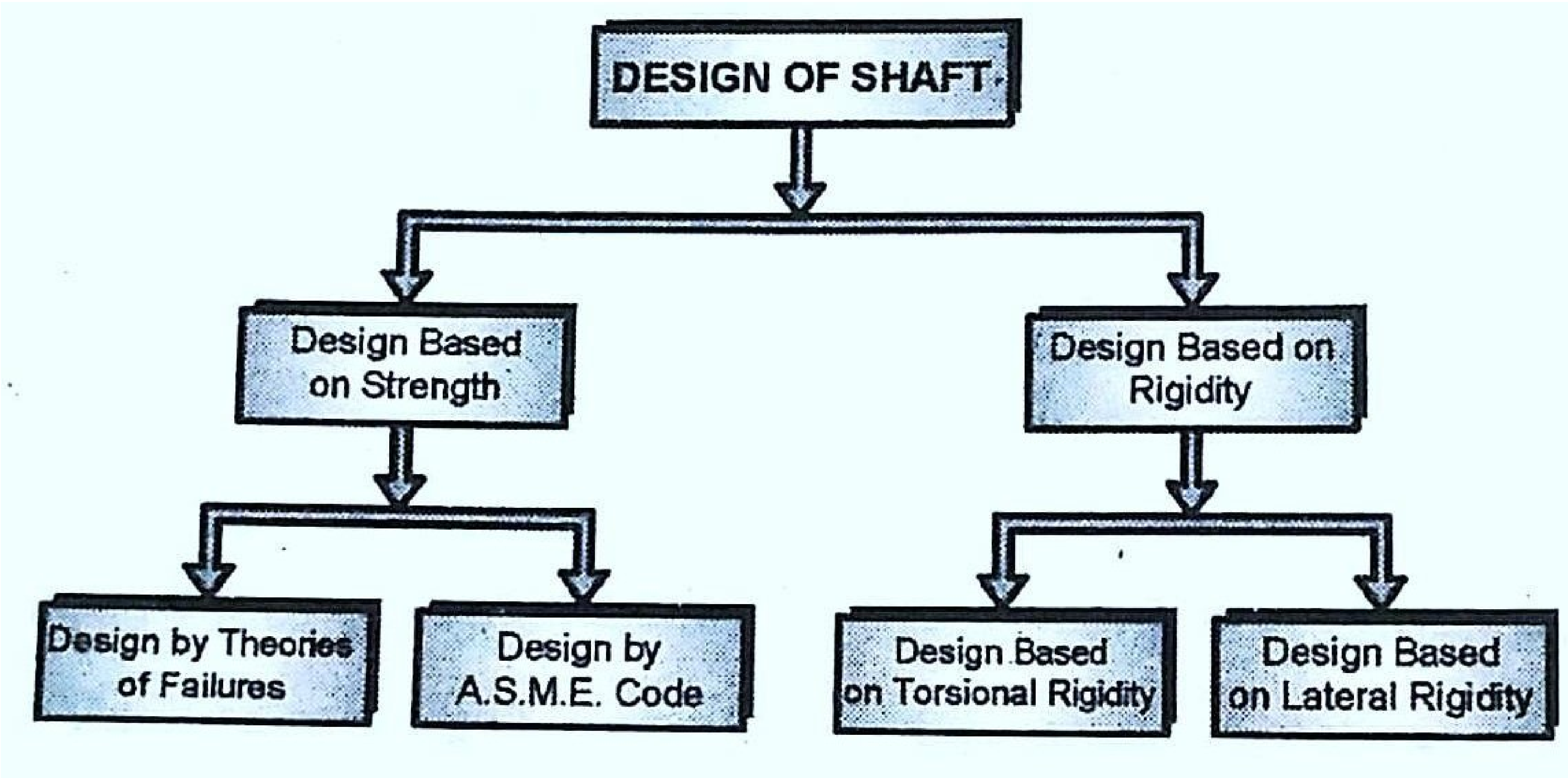
$$\sigma_b = \frac{32\,Mb}{[\pi d^3]}$$

Torsional shear stress

$$r = \frac{16\,T}{[\pi d^3]}$$

When the shaft is subjected to combination of loads, the principal stress and principal shear stress are obtained by constructing Mohr's circle

Design of shaft



Design of shaft on Strength Basis

1. Design of shaft by theories of failures
2. Design of shaft by A.S.M.E. Code

Design Procedure of shaft

Generally the shafts are subjected to torsion as well as bending

Basic bending equation is : $\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$

Considering the first two terms $\frac{M}{I} = \frac{\sigma}{Y}$ one can write $\sigma = \frac{MY}{I}$

Design of shaft on Strength Basis

$$\sigma = \frac{MY}{I}$$

Now M is M_b Bending Moment

Y is d/2

and

$$I = \frac{\pi}{64}(d^4)$$

Therefore

$$\sigma = \frac{32\ M_b}{\pi d^3}$$

Design of shaft on Strength Basis

Basic Torsion equation is : $\frac{T}{J} = \frac{Fs}{R} = \frac{G\theta}{l}$

Now T is M_t Torsional Moment

Now Fs is τ shear stress

Considering the first two terms $\frac{Mt}{J} = \frac{\quad}{R}$ one can write $= \frac{Mt \;\; R}{J}$

Design of shaft on Strength Basis

$$= \frac{Mt \quad R}{J}$$

Now M_t Torsional Moment or Torque

R is d/2

and

$$J = \frac{\pi}{32}(d^4)$$

Therefore $$= \frac{16 \quad M_t}{\pi d^3}$$

Design of shaft on Strength Basis

a] Maximum Principal Stress Theory: The shaft is subjected to bending and torsional moments without any axial force.

$$\sigma_x = \sigma_b = \frac{32 M_b}{\pi d^3}$$

$$\tau = \frac{16 M_t}{\pi d^3}$$

$$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{(M_b)^2 + (M_t)^2}\right]$$

Permissible value of max. principal stress $\sigma_1 = \frac{S_{yt}}{(fs)}$

- These equations are used to determine shaft diameter on the basis of principal stress theory.
- The maximum principal stress theory gives good predictions for brittle materials.
- Shafts are made of ductile material like steel and therefore, this theory is not applicable to shaft design.

1.Shaft Design by theories of failures

b] Maximum Shear Stress Theory: the shaft is subjected to bending and torsional moments

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2}$$

so

$$\tau_{max.} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2}$$

Permissible value of max. shear stress

$$\tau_{max.} = \frac{S_{sy}}{(fs)} = \frac{0.5S_{yt}}{(fs)}$$

These equations are used to determine shaft diameter on the basis of maximum shear stress theory.

The maximum shear stress theory is applicable to ductile materials. Since the shafts are made of ductile materials, it is more logical to apply this theory to shaft design.

Design of shaft on Strength Basis

c] Distortion Energy Theory: The shaft is subjected to bending and torsional moments

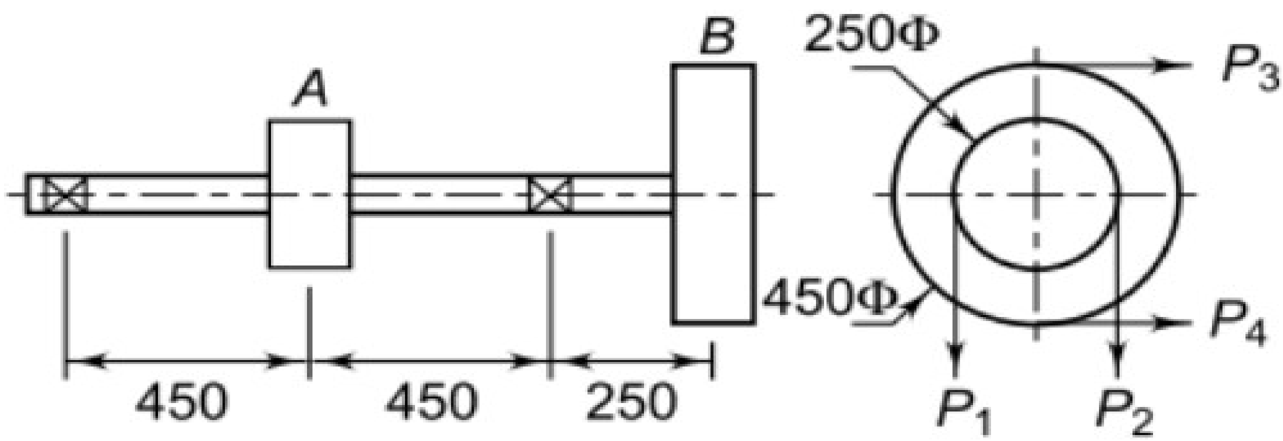
$$\sigma_V = \frac{32}{\pi d^3}\left[\sqrt{(K_b M)^2 + 0.75\,(K_t T)^2}\right]$$

$$\sigma_1 = \frac{S_{yt}}{(fs)}$$

- These equations are used to determine shaft diameter on the basis of distortion energy theory.
- The maximum shear stress theory is applicable to ductile materials. The design of shaft by distortion energy theory is very accurate. Hence distortion energy theory is most widely theory used for shaft design.

NUMERICAL

A line shaft supporting two pulleys A and B as shown. Power is supplied to the shaft by means of a vertical belt on the pulley A, which is then transmitted to the pulley B carrying a horizontal belt. The ratio of belt tension on tight and loose sides is 3:1. The limiting value of tension in the belts is 2.7 kN. The shaft is made of plain carbon steel 40C8 (S_{ut} = 650 N/mm^2 and S_{yt} = 380 N/mm^2). The pulleys are keyed to the shaft. Determine the diameter of the shaft according to the ASME code if, k_b = 1.5 and k_t = 1.0



Given data

S_{ut} = 650 N/mm^2 S_{yt} = 380 N/mm^2

For Belt tensions the ratio T_1/T_2= 3

Maximum belt tension : 2.7 kN = 2700 N

Combined shock and fatigue factor in bending and torsion K_b = 1.5 & K_t = 1.0

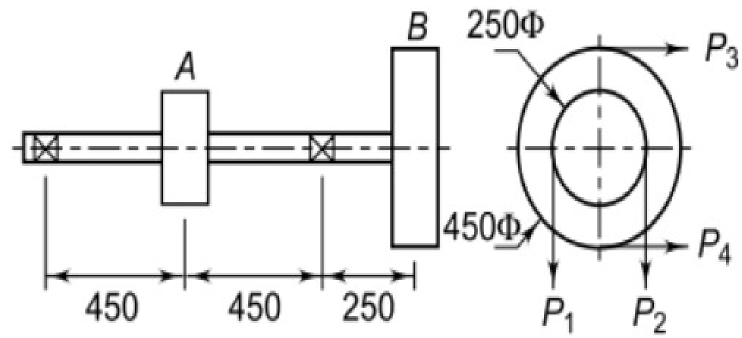
Permissible shear stress, τ_{perm} = Minimum of [0.75 (0.30 S_{yt})]
or
[0.75 (0.18 S_{ut})]

τ_{perm} = Minimum of [0.75 (0.30 x 380)]
or
[0.75 (0.18 x 650)]

τ_{perm} = Minimum of [85.5)] or [87.75]

There fore τ_{perm} = 85.5 N/mm^2

$(T_1-T_2)\frac{250}{2} = (T_3-T_4)\frac{450}{2}$ Therefore $(T_1-T_2) \times 250 = (T_3-T_4) \times 450$



but $T_1= 3T_2$ $T_3= 3T_4$

$(T_1- \frac{T1}{3}) \times 25 = (T_3 - \frac{T3}{3}) \times 45$ ➡ $(\frac{3T1-T1}{3}) \times 25 = (\frac{3T3-T3}{3}) \times 45$

$(\frac{2T1}{3}) \times 25 = (\frac{2T3}{3}) \times 45$ ➡ $T_1 \times 25 = T_3 \times 45$ ➡ $T_1 = T_3 \times \frac{45}{25}$ ➡ $T_1 = 1.8 \times T_3$ ➡ $T_3 = \frac{T1}{1.8}$

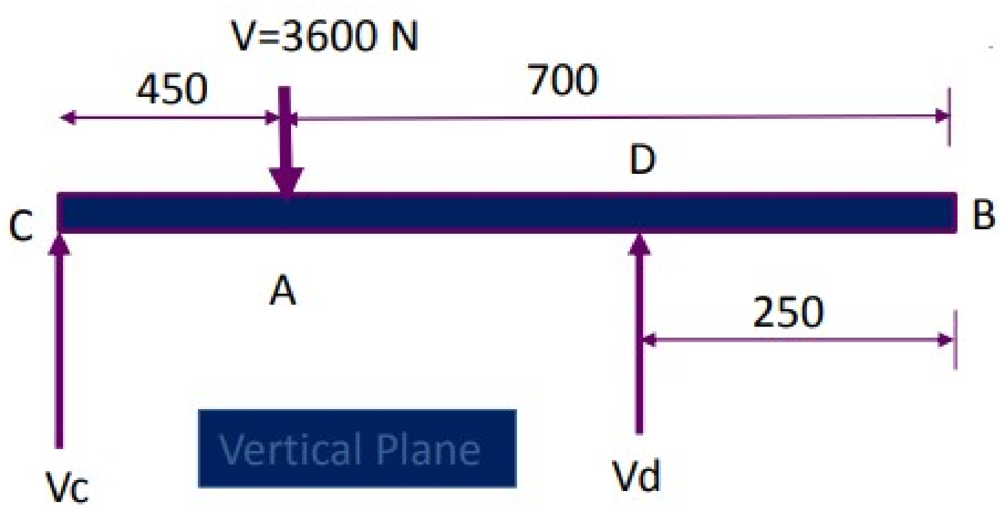
That means Maximum tension will be T_1 i.e. tight side of belt on smaller pulley T_1= 2700 N

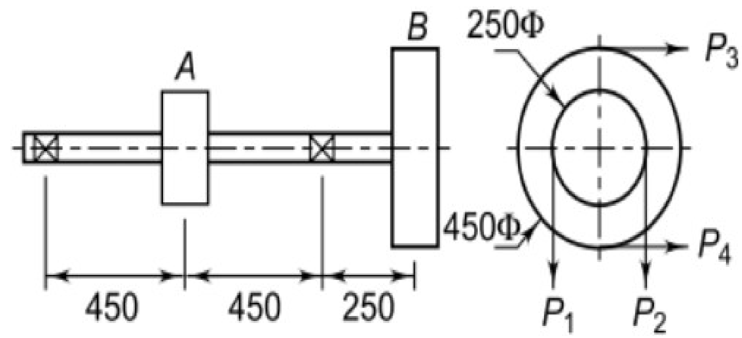
Therefore $T_3 = \frac{2700}{1.8}$ ➡ T_3= 1500 But $T_2 = \frac{T1}{3}$ & $T_4 = \frac{T3}{3}$

Therefore $T_2 = \frac{2700}{3} = 900$ & $T_4 = \frac{1500}{3} = 500$

Vertical Force Analysis

$$V = (T_1+T_2) = (2700 + 900) = 3600\ N$$





Now let us apply law of equilibrium of moments

$$\Sigma (M_b)_c = 0 \quad (+ve)$$

$+ \ Vd * 900 - V * 450 = 0$

$Vd = 1800\ N$

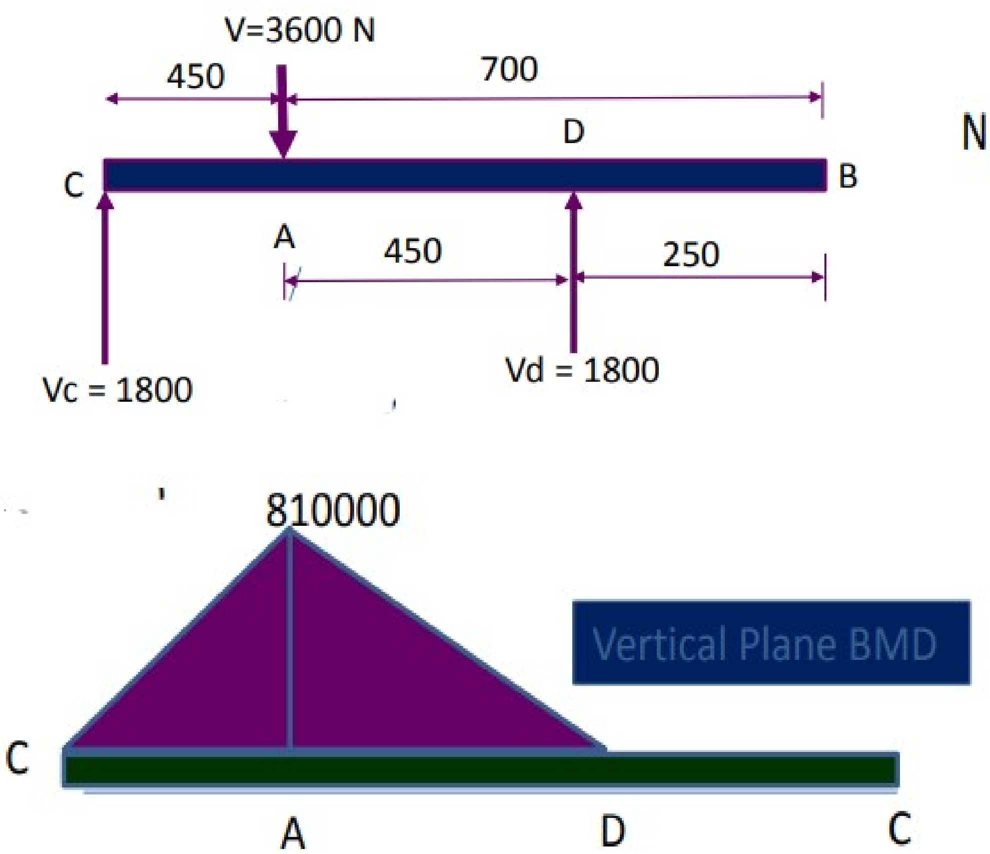
Now letus apply law of equilibrium of forces

$$\Sigma F = 0 \quad (+ve \downarrow)$$

$-Vd + 3600 - Vc = 0$

$Vc = 1800\ N$

Vertical Force Analysis



Now let us find bending moments and draw a BM diagram

BM at point B = 0 N-mm

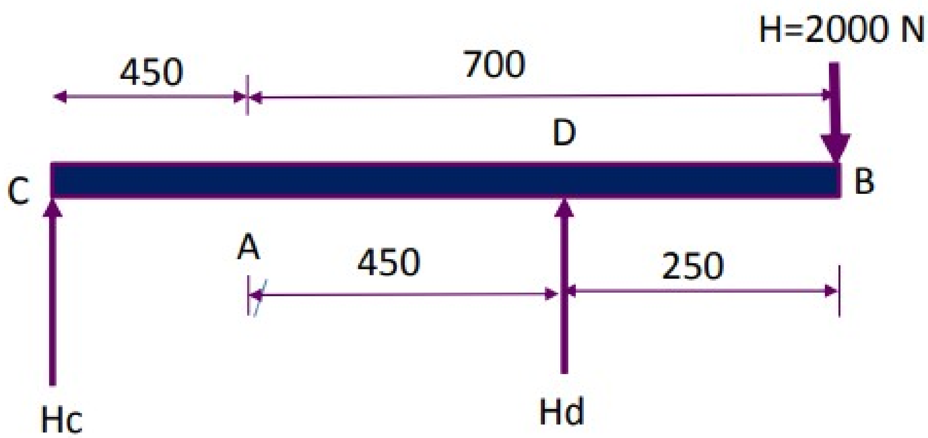
BM at point D = 0 N-mm

BM at point A = Vd * 450 = 810000 N-mm

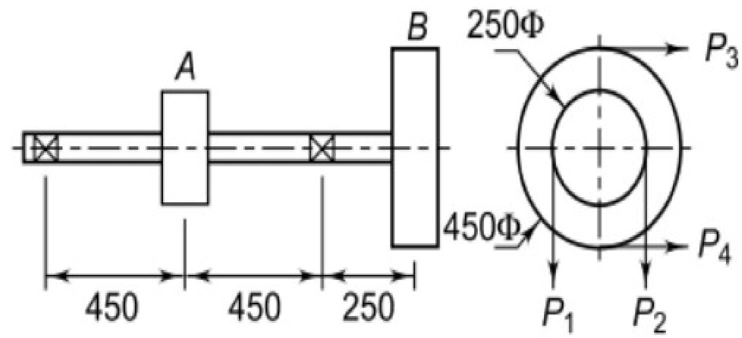
BM at point C = 0 N-mm

Horizontal Force Analysis

$$H = (T_3+T_4) = (1500 + 500) = 2000\ N$$



Horizontal Plane



Now let us apply law of equilibrium of moments

+ve

$$\Sigma (M_b)_c = 0$$

$- H * 1150 + Hd * 900 = 0$

$Hd = 2556\ N$

Now letus apply law of equilibrium of forces

$$\Sigma F = 0$$

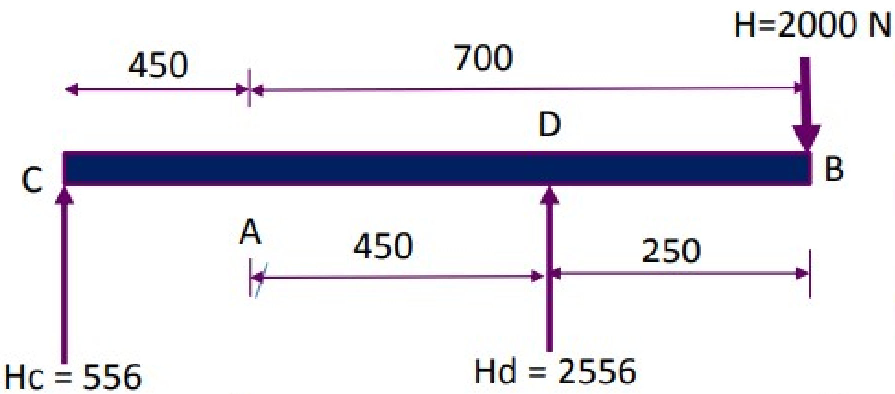
+ve

$H - Hd + Hc = 0$

$Hc = 556\ N$

Horizontal Force Analysis

Now let us find bending moments and draw a BM diagram

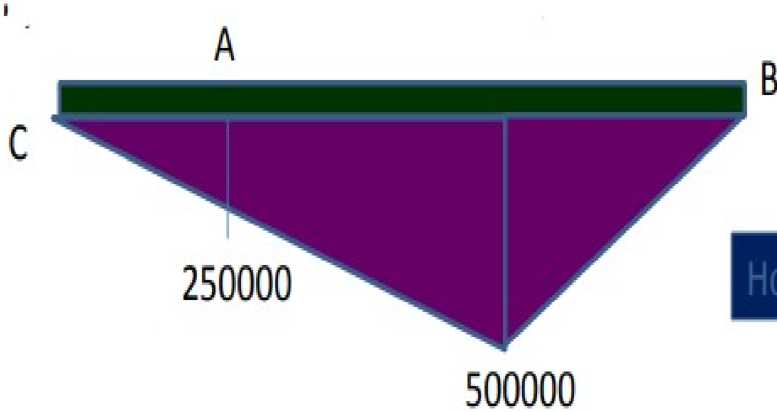


BM at point B = 0 N-mm

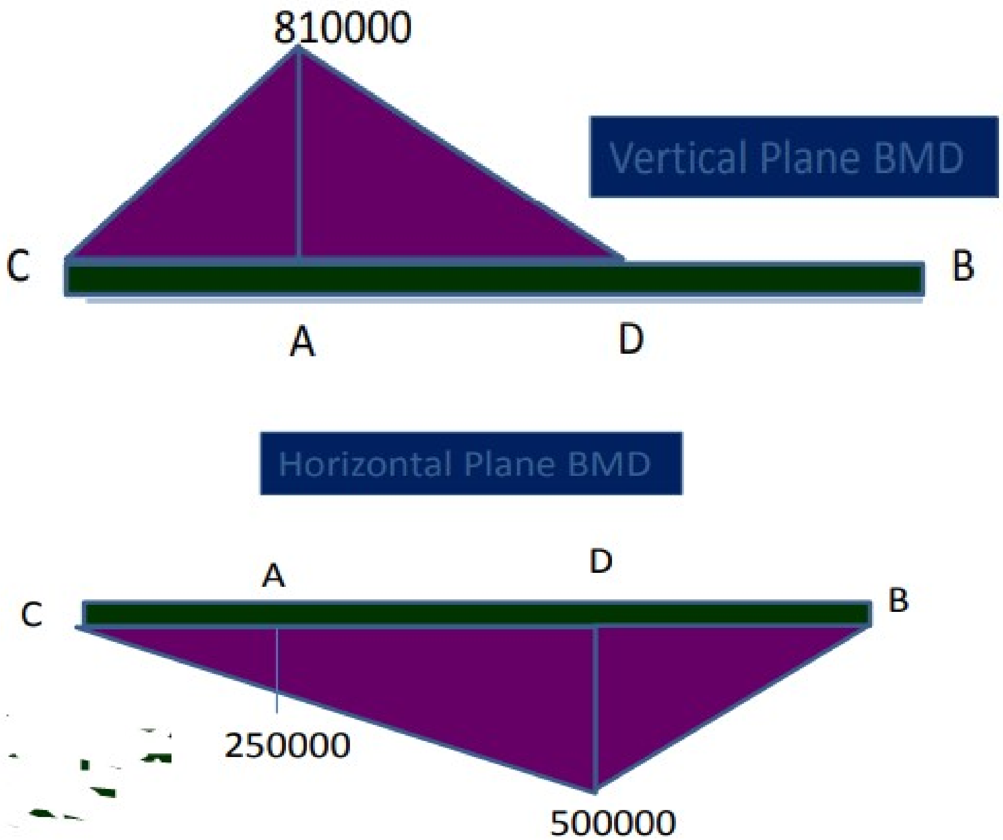
BM at point D = - H x 250 = -500000 N-mm

BM at point A = - H x 700 + Hd x 450 = -250000 N-mm

BM at point C = - H x 1150 + 2556 x 900 = 0 N-mm



Horizontal Plane BMD



Now let us find Resultant bending moments by performing vector sum of Bending Moments in Vertical and Horizontal Plane

Resultant BM at point B = **0 Nmm**
Resultant BM at point D = **500000 Nmm**
Resultant BM at point A = **847703 Nmm**
Resultant BM at point C = **0 Nmm**

Maximum Resultant Bending Moment $(M_b)_{max}$ = 847703 N-mm

Torque experienced by the shaft, $M_t = (T_1 - T_2) \times R$

$$= (2700 - 900) \times \left(\frac{250}{2}\right) = 225000 \text{ N-mm}$$

Now Applying ASME Code $$d^3 = \frac{16}{\pi \tau_{perm}} \sqrt{(K_b \cdot M_b)^2 + (K_t \cdot M_t)^2}$$

Where Combined shock and fatigue factor in bending and torsion $K_b = 1.5$ & $K_t = 1.0$

Also already we have calculated τ_{perm} = 85.5 N/mm^2

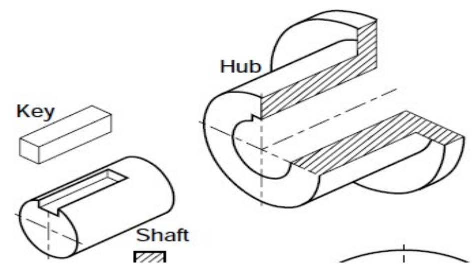
Therefore $$d^3 = \frac{16}{\pi * 85.5} \sqrt{(1.5 * 847703)^2 + (1.0 * 225000)^2}$$

Therefore $d = 42.53 \approx 45$ mm

Shaft Design by A.S.M.E. CODE

A.S.M.E. code used for design of shaft is based on maximum shear stress theory of failure.

$$\tau_{max.} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (M_t)^2}$$



According to A.S.M.E. code the values of allowable shear stress are as follows;
[without keyway] take minimum of two values

$$\tau_{all} = 0.3\ S_{yt} \text{ or } 0.18\ S_{ut}$$

According to A.S.M.E. code the values of allowable shear stress are as follows;
[with keyway] take minimum of two values

$$\tau_{all} = 0.75 \times 0.3S_{yt} \text{ or } 0.75 \times 0.18S_{ut}$$

Note: keyway on the shaft reduces the strength of the shaft. This is due to stress concentration near corners of keyway.

Shaft Design by A.S.M.E. CODE

ASME Code of Shaft Design : $d^3 = \frac{16}{\pi\ \tau_{perm}} \sqrt{(K_b \cdot M_b)^2 + (K_t \cdot M_t)^2}$

where

d is the diameter of the shaft

τ_{perm} is allowable shear stress or shear strength of the shaft material

M_b is Maximum bending moment to which the shaft is subjected

M_t is Maximum torsional moment or the torque to which the shaft is subjected

K_b & K_t are the combined shock and fatigue factor in bending and torsion

Shaft Design by A.S.M.E. CODE

ASME Code of Shaft Design : $d^3 = \frac{16}{\pi \tau_{perm}} \sqrt{(K_b \cdot M_b)^2 + (K_t \cdot M_t)^2}$

where

τ_{perm} is allowable shear stress or shear strength of the shaft material and according to ASME Code its value is selected as minimum of the following two values

$(0.18 \cdot S_{ut})$ and $(0.30 \cdot S_{yt})$

where S_{ut} is ultimate tensile strength

S_{yt} is yield point tensile strength

Shaft Design by A.S.M.E. CODE

ASME Code of Shaft Design : $d^3 = \frac{16}{\pi \tau_{perm}} \sqrt{(K_b \cdot M_b)^2 + (K_t \cdot M_t)^2}$

Usually the shafts are mounted with rotational members like pulleys, gears, flywheels, etc. and these rotational members are mounted over the shafts with the help of keys. Therefore it is necessary to provide a keyway on the shaft. Provision of keyway on the shaft results in weakening of the shaft. To take into account weakening of the shaft (loss in strength of the shaft), the shaft strength τ_{perm} is reduced to 75%

Therefore in reality the permissible shear stress for shaft material or the shaft strength τ_{perm} is assumed as minimum of the following two values

$(0.75) \cdot (0.18 \cdot S_{ut})$ & $(0.75) \cdot (0.30 \cdot S_{yt})$

Shaft Design by A.S.M.E. CODE

Why ASME Code of Shaft Design is important or generally preferred by design engineers

- It is based on Maximum shear stress theory of failure.
- Proper selection of factor of safety is made.
- Takes into account column action factor.
- Takes into account shock and fatigue load.
- Takes into account the axial forces.
- With certain modification the same code can be employed for designing of hollow shafts.

Column action factor: This arises due the phenomenon of buckling of long slender members (here shaft) which are acted upon by axial compressive loads. The net normal stress can be either positive or negative.

Values of shock and fatigue factors k_b and k_t

Application	k_b	k_t
(i) Load gradually applied	1.5	1.0
(ii) Load suddenly applied (minor shock)	1.5–2.0	1.0–1.5
(iii) Load suddenly applied (heavy shock)	2.0–3.0	1.5–3.0

Design of shaft based on Torsional Rigidity

Torsional rigidity is defined as " torque required to produce a torsional deflection or an angle of twist of one radian in the shaft."

A transmission shaft is said to be rigid on the basis of torsional rigidity, if it does not twist too much under the action of an external torque.

$$\theta_r = \frac{M_t l}{JG}$$

where,

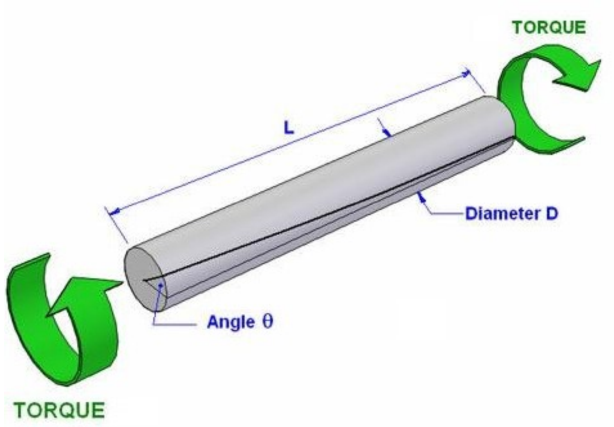
θ = angle of twist (deg.)

l = length of shaft subjected to twisting moment (mm)

M_t = torsional moment (N-mm)

G = modulus of rigidity (N/mm^2)

d = shaft diameter (mm)



- This equation is used to design the shaft on the basis of torsional rigidity.
- The permissible angle of twist for machine tool applications is 0.25° per metre length. For line shafts, 3° per metre length is the limiting value. Modulus of rigidity for steel is 79 300 N/mm^2 or approx. 80 kN/mm^2

Design of shaft based on Lateral Rigidity

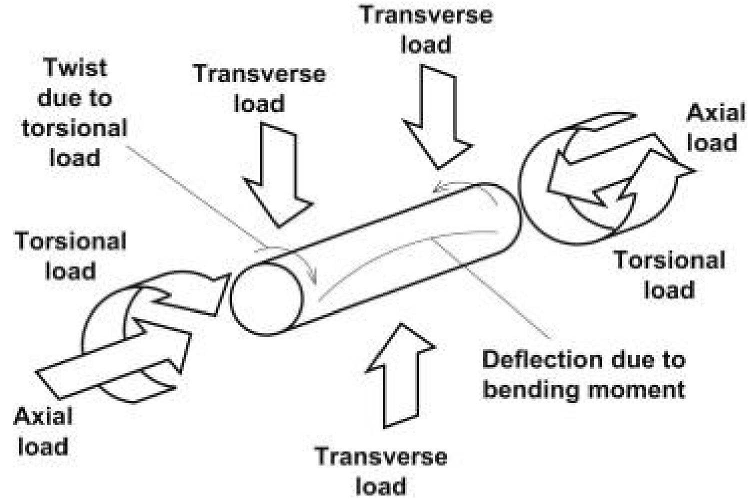
- Lateral rigidity of the shaft at given location is "the lateral force required to produce a lateral deflection of one unit".
- A transmission shaft is said to be rigid on the basis of lateral rigidity, if it does not deflect too much under the action of an external forces and bending moment.

$$\delta = 0.001\, L \text{ to } 0.003\, L$$

This equation is used to design the shaft on the basis of lateral rigidity.

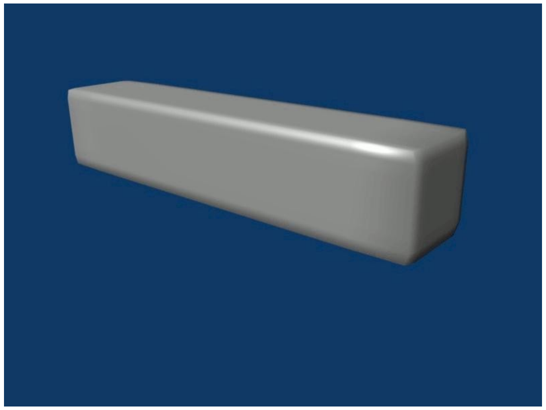
e.g. for cantilever beam maximum deflection is given by,

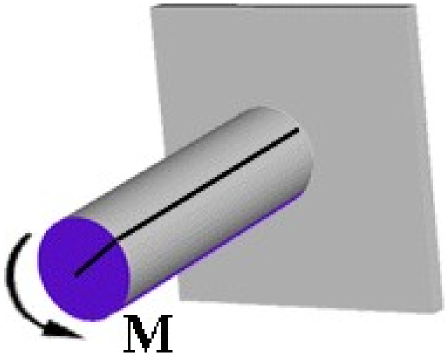
$$\delta = \frac{FL}{3\,EI}$$



Design of shaft

Torsional Rigidity





Lateral Rigidity