Springs

Spring is an elastic machine element or resilient member, which deflects under the action of the load and returns to its original shape when the load is removed

Induces torsional shear stresses in spring wire even the spring is subjected to axial loading

Types

Compression Spring

Extension Spring

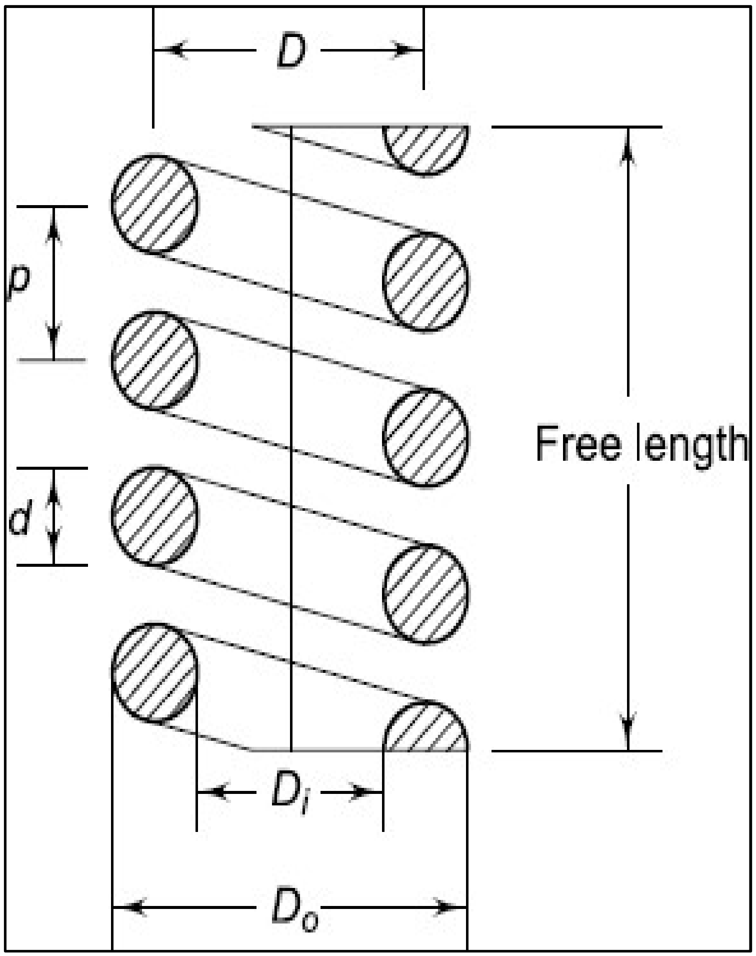
Torsion Spring

Leaf Spring

Function and Uses of Springs

- To apply forces, & controlling motions as found in brakes & clutches.
- To measure spring forces as in a spring balance measuring instrument.
- To store energy as in clocks, springs, toys.
- To produce cushioning effect or to reduce the effect of impact or shock loading as in a vehicle suspension springs, railway wagon springs.
- To provide vibration isolation.
- To control motion by maintaining contact b/w two elements. E.g. Cam and followers

Terminology of Springs



Mean diameter

$$D = \frac{D_i + D_o}{2} \quad \text{Eq 1.1}$$

Spring index parameter

$$C = \frac{D}{d} \quad \text{Eq 1.2}$$

d is the diameter of spring wire;

D is the diameter of spring or mean coil diameter

D_o is the outer diameter of spring = D +d

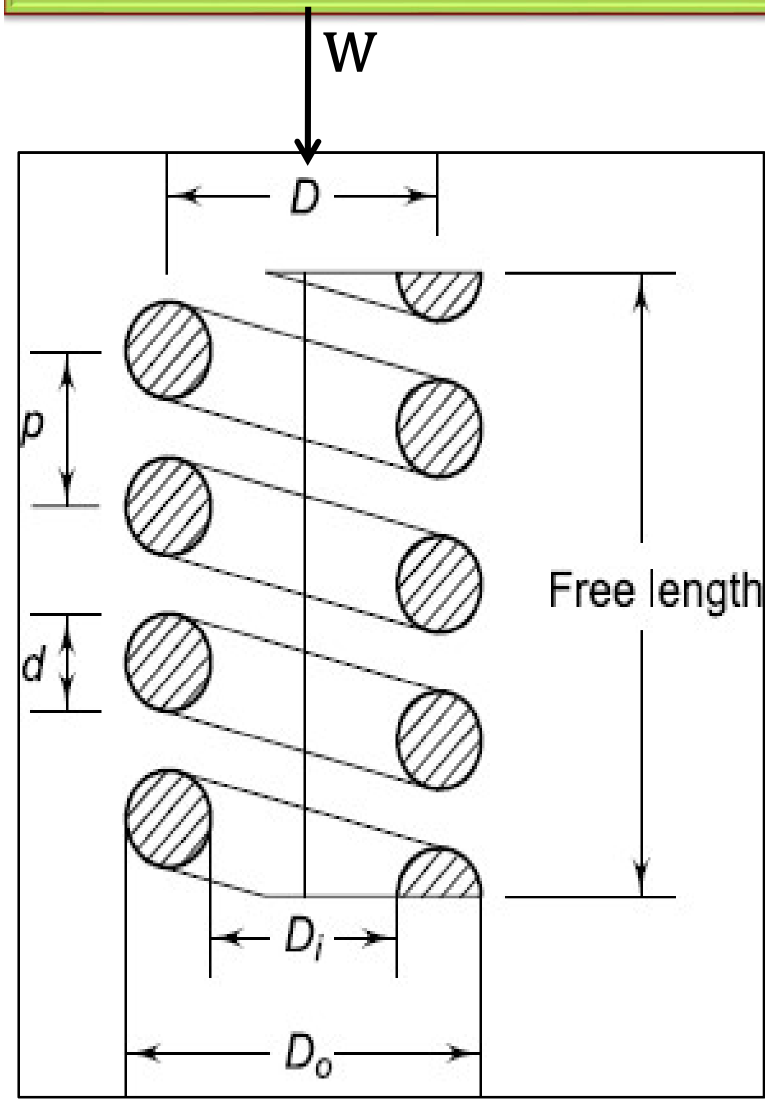
D_i is the inner diameter of spring = D -d

If C low value: Stresses excessive due to curvature effect

If C high: Prone to buckling

C: 6-9 is preferred

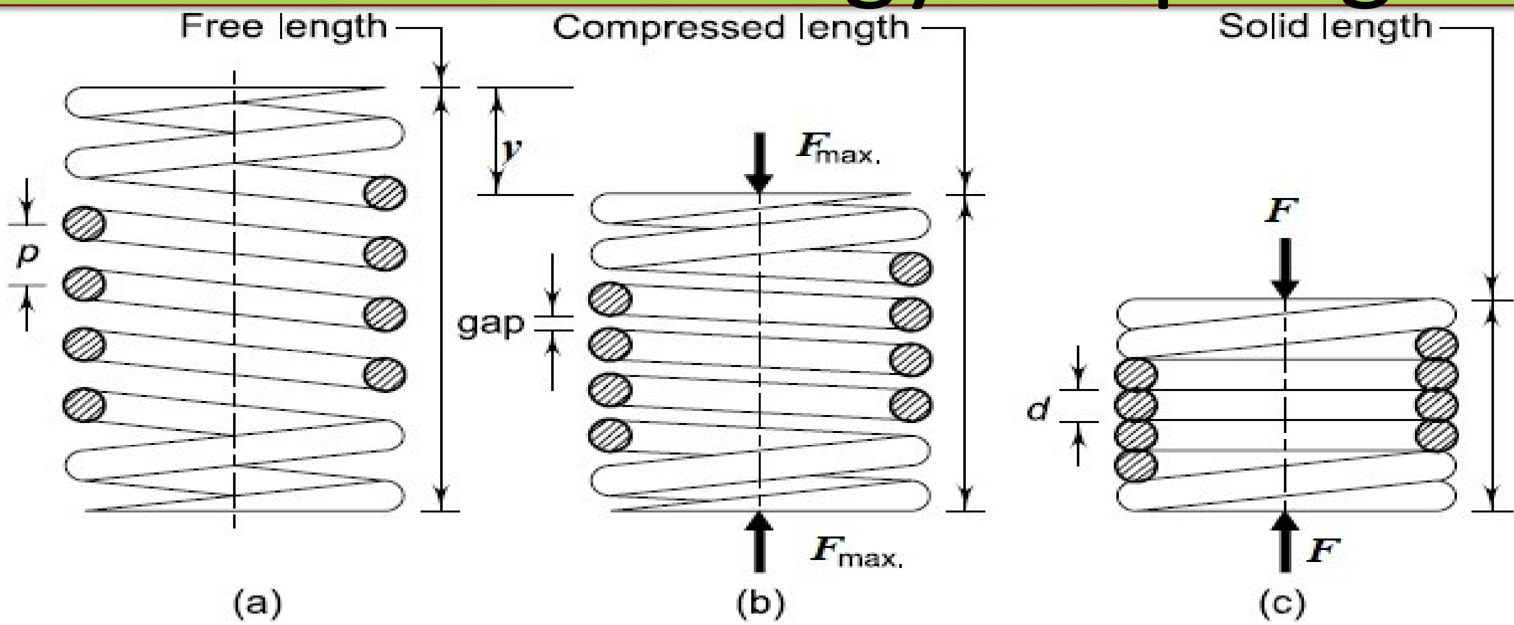
Terminology of Springs



- With respect to spring wire, load **F** acts as eccentric transverse shear load while with respect to spring it is axial compressive load.
- Spring wire is subjected to bending stress

- **Active (N) and Inactive coils**: Active coils are the coils in the spring which contribute to spring action, support the external force and deflect under the action of force.
- End coils which do not contribute to spring action are called inactive coils. No. of inactive coils is generally 2
- N_t denotes total no. of coils.

Terminology of Springs



Free length(L_{free}): Length of spring when there is no load acting or manufactured length

Compressed length (L_{comp}): Length of spring under maximum deflected condition (gap between coils ≠ 0)

Solid Length (L_{solid}): Length of spring when gap between coils is zero

$$L_{solid} = N_t d$$

Eq 1.3

$$L_{free} = L_{comp} + Y_{max}$$
$$= L_{solid} + 1.15Y_{max}$$

Eq 1.4

where $L_{comp} = L_{solid}$ + total gap between coils under maximum compressed condition i.e. 15% of Y_{max}

Terminology of Springs

Pitch of the coil (p): Axial distance between adjacent coils in uncompressed state of spring

$$p = \frac{Free\ Length}{(\mathrm{N_t} - 1)}$$

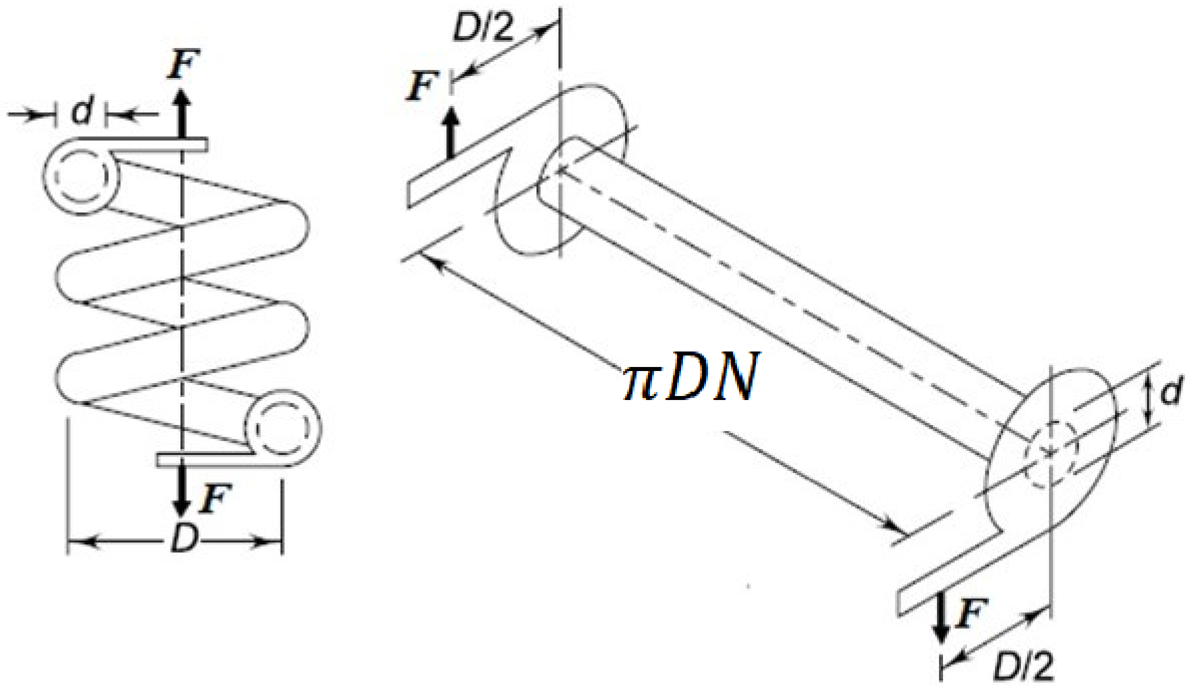
Eq 1.5

Spring Stiffness or spring rate (k): Force required to produce unit deflection

$$k = \frac{W}{Y_{max}}$$

Eq 1.6

Unbent Spring



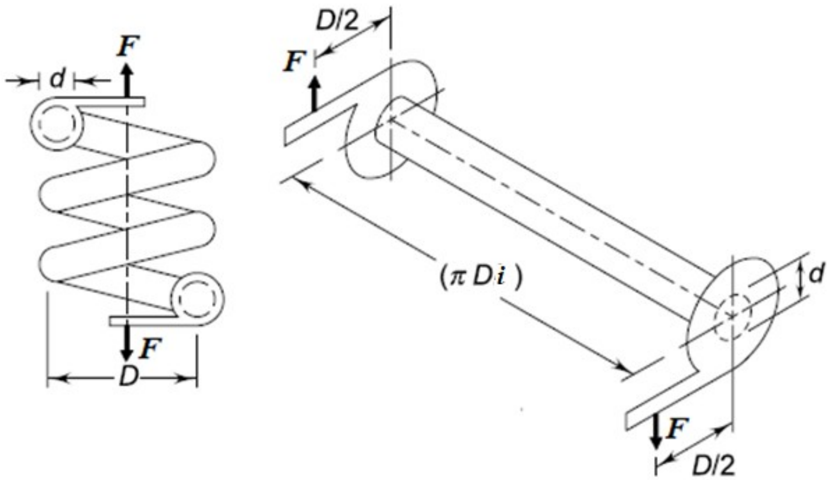
Two basic equations for the design of helical springs

- Load-stress equation
- Load-deflection equation

Dimensions

- The diameter of the bar is equal to the wire diameter of the spring (d)
- The length of the equivalent bar is πDN
- Bar is fitted with bracket of length (D/2)

Unbent Spring



Torsional moment due to force F

$$M_t = \frac{FD}{2}$$

Torsional shear stress due to the force F

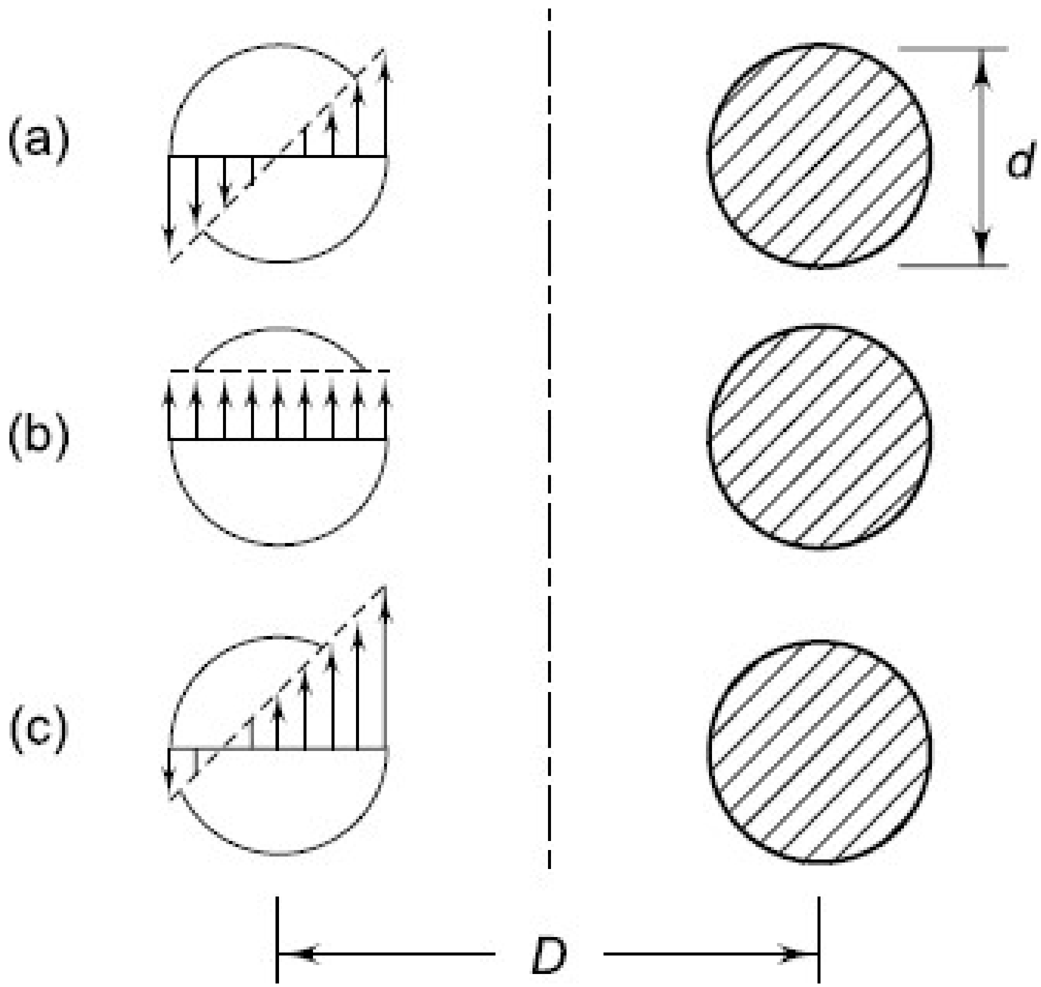
$$\tau_1 = \frac{Tr}{J} = \frac{G\theta}{l} = \frac{8FD}{\pi d^3}$$

Eqn 1.7

Considering equivalent bar in the form of helical coil, the additional stresses accounted

- Direct or transverse shear stress in spring wire
- Length of inside fibre is less than length of outside fibre which induces stress concentration at the inside of fibre coil

Stresses in spring wire



Pure torsional stress

Direct shear stress

Combined torsional, direct and curvature shear stress

Load-Stress Equation

Modification in the stress equation to accommodate the direct shear and curvature stress effect. **Wahl's factor (K_w)** is used in the design of helical spring to consider the effect of direct shear stress and curvature effect on maximum shear stress induced in the wire

$K_w = K_s K_c$ where, K_s is shear stress factor & K_c is curvature factor

Direct shear stress, $\tau_2 = \frac{4F}{\pi d^2} = \frac{8FD}{\pi d^3}\left(\frac{0.5d}{D}\right)$

$$\tau = \tau_1 + \tau_2 = \frac{8FD}{\pi d^3}\left(1 + \frac{0.5d}{D}\right) = K_s \frac{8FD}{\pi d^3}$$

Combined torsional, direct and curvature shear stress by Wahl

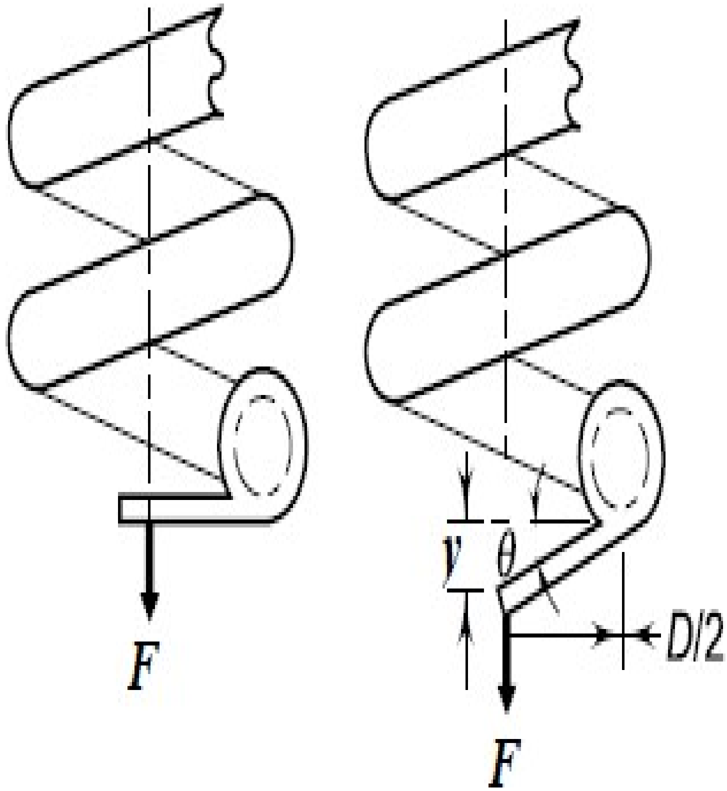
$$\tau = K_w \frac{8FD}{\pi d^3}$$ Eqn 1.8

Wahl factor

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$ Eq 1.9

LOAD-STRESS EQUATION

Load-Deflection (F-Y) Equation



U = Strain Energy stored in the spring wire due to Torsional moment

$$U = \frac{1}{2} \times M_t \times \theta = \frac{1}{2} M_t \times \frac{M_t L}{GJ} = \frac{1}{2}\left(\frac{FD}{2}\right)^2 \times \frac{\pi D N}{G \times \frac{\pi d^4}{32}}$$

$$\boxed{U = \frac{4F^2 D^3 N}{G d^4}}$$ Eq 1.10

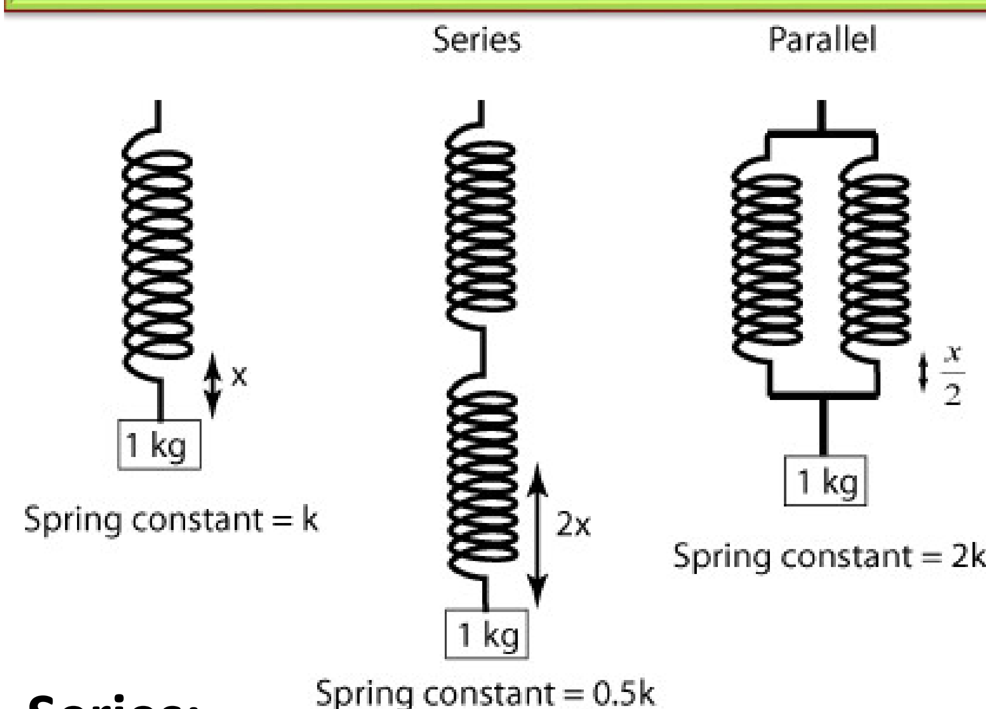
By Using Castigliano's Theorem

$$y = \frac{\partial U}{\partial W} = \frac{\partial}{\partial F}\left[\frac{4F^2 D^3 N}{G d^4}\right]$$

$$\boxed{y = \frac{8 F C^3 N}{G d}}$$ Eq 1.11

$$\boxed{K = \frac{F}{y} = \frac{G d}{8 C^3 N}}$$ Eq 1.12

Series and Parallel Connections



Parallel:

- Force acting is sum of forces acting on individual spring ($F = F_1 + F_2$)
- Total deflection will be same (δ)

$$y_1 k_1 = F \qquad y_2 k_2 = F$$

$$\boxed{k = k_1 + k_2 + ..}$$

Eq 1.13

Series:

- Force acting is same (F)
- Total deflection will be sum of deflections of individual springs $y = y_1 + y_2$

$$y_1 = \frac{F}{k_1} \qquad y_2 = \frac{F}{k_2}$$

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + ..}$$

Eq 1.14

Design of Closed Coil Helical Spring

- Objectives for designing of closed coil helical spring are :

  1. It should possess sufficient strength to withstand the external load: Use Load-Stress equation
  2. Have required Load-Deflection characteristics: Use Load Deflection Equation
  3. It should not buckle under the external load

Note: The main dimensions to be calculated in the spring design are Spring wire diameter (d), mean coil diameter (D) and the number of active coils (N). The first two are calculated by the load-stress equation, while the third is calculated by the load-deflection equation.

Use of low FOS in closed Coil Helical Spring

- **Factor of Safety:** The FOS in the design of springs is usually 1.5 or less. The use of a relatively low FOS is justified on the following grounds:

1. In most of the applications, springs operate with well defined deflections. Therefore, the forces acting on the spring and corresponding stresses can be precisely calculated. It is not necessary to take higher factor of safety to account for uncertainty in external forces acting on the spring.
2. In case of helical compression springs, an overload will simply close up the gaps between coils without a dangerous increase in deflection and stresses.
3. In case of helical extension springs, usually overload stops are provided to prevent excessive deflection and stresses.
4. The spring material is carefully controlled at all stages of manufacturing. The thin and uniform wire cross-section permits uniform heat treatment and cold working of the entire spring.

Table 1

Type of ends	*Number of active turns (N)*
Plain ends	N_t
Plain ends (ground)	$\left(N_t - \frac{1}{2}\right)$
Square ends	$(N_t - 2)$
Square ends (ground)	$(N_t - 2)$

Helical Spring: Design Procedure

- Estimate spring force (F) and required deflection (y), In some cases it will be specified
- Select suitable material, Obtain permissible shear stress, $\tau = 0.5S_{ut}$
- Assume spring index value C, Preferred (8-10)
- Calculate Wahl factor, K_w (Eqn. 1.9)
- Determine wire diameter from load-stress equation, d (Eqn 1.8)
- Determine coil diameter, D, (eqn 1.2)
- Determine number of active coils, N by load deflection equation, N (eqn 1.11)
- Determine total number of coils, N_t, (table 1)
- Determine solid length of the spring, (Eqn 1.3)
- Find actual deflection, y by load-deflection equation (eqn 1.11) and also free length (Assuming suitable gaps)
- Obtain pitch of the coil, p (Eqn 1.5)
- Determine actual spring rate,K(Eqn 1.12)

Helical Spring: Design Procedure

- A helical compression spring that is too long compared to the mean coil diameter (D), acts as a flexible column and may buckle at a comparatively low axial force. The spring should be preferably designed as buckle-proof.
- Compression springs, which cannot be designed buckle-proof, must be guided in a sleeve or over an arbor.
- Therefore last step is to check for buckling in springs

$$\frac{L_{free}}{D} \leq 3.5$$

$$\Rightarrow Spring\ is\ safe\ with\ respect\ to\ buckling\ failure$$

$$\frac{L_{free}}{D} > 3.5$$

$$\Rightarrow Buckling\ occurs, then\ spring\ should\ be\ guided\ in\ a\ sleeve\ to\ prevent\ buckling\ failure$$