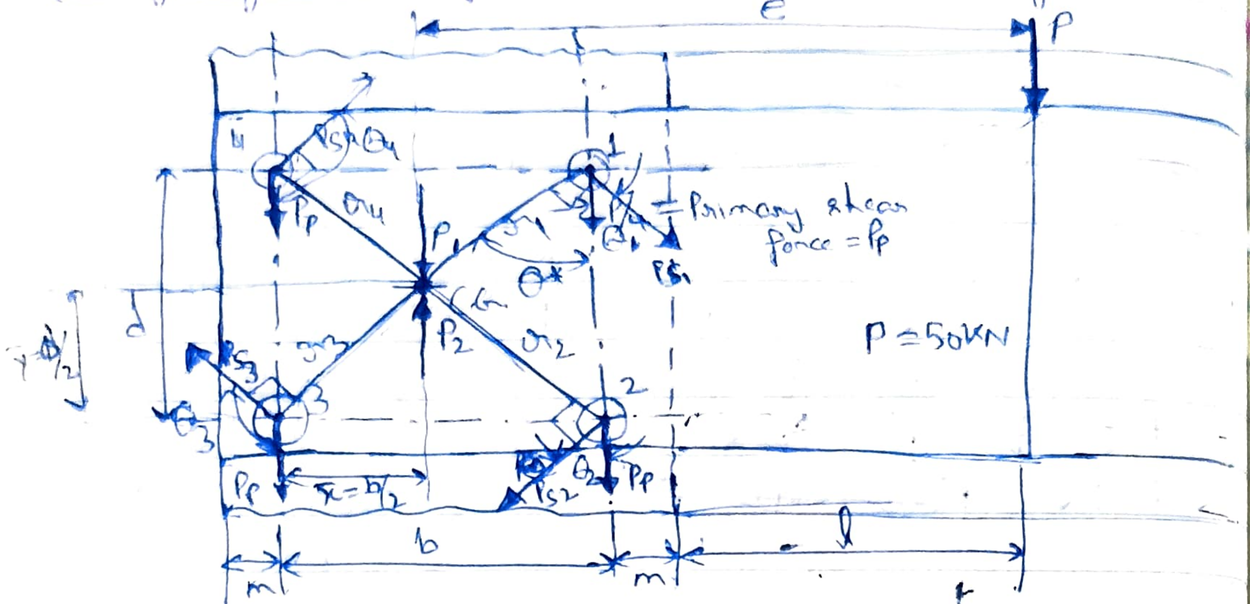
Design of Riveted joint under eccentric loading



(1) Determination of C.G. of group of rivets

$$\bar{x} = b/2 \; ; \; \bar{y} = d/2$$

(2) Introduce two equal & opposite forces (P_1 & P_2) through CG of group of rivets as shown above

$$P_1 = P_2 = P$$

(3) Eccentricity (e)

$$e = l + m + b/2$$

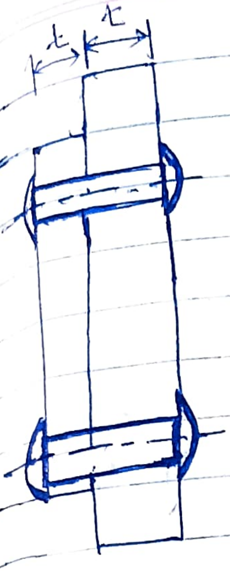
(4) Effect of P_1 :- is to cause a primary shear force (P_p) of equal magnitute at each & every rivet as shown in the figure,

$$P_p = \frac{P_1}{n} = \frac{P}{4}$$

(5) Effect of P_1 & P_2 :- P & P_2 causes a twisting moment of $(P \times e)$ in the cw direction with respect to group of rivets.

- Due to this twisting moment, rivets are subjected to a secondary shear force (P_s).

$$T.M. = P \times e \quad (2)$$



$\theta_1 = 90 - \theta^*$

~~Tmax~~ $\tan\theta^* = \frac{b/2}{d/2}$

θ → angle b/w P_p & P_s

$P = 50\,kN$
$d = 200, b = 100$
$l = 100, m = 20\,mm$
$\tau_{per} = 75\,MPa$

- Secondary shear force at any rivet is directly proportional to its distance from CG of group of rivets (r).
- Secondary ~~s~~ shear force is maximum at a rivet which is far away from the CG of group of rivets
- Secondary shear force at all the rivets are equal in magnitude when all the rivets are equidistance from the CG of group of rivets (i.e. $r_1 = r_2 = r_3$ ------)
- Secondary SF always act ⊥ to to line joining CG of group of rivets & centroid of the rivets.

(6) Values of r:-

$r_1 = r_2 = r_3 = r_4 = r = \sqrt{(b/2)^2 + (d/2)^2}$

$P_{s1} = P_{s2} = P_{s3} = P_{s4}$ $(\because P_s \propto r)$

$P_s = kr_1$, $P_s = kr_2$, $P_s = kr_3$

(7) Magnitudes of P_{s1}, P_{s2}, ------

$P_{s1}r_1 + P_{s2}r_2 + P_{s3}r_3 + P_{s4}r_4 = P \times e$

$k[r_1^2 + r_2^2 + r_3^2 + \ldots] = Pe$

$\frac{P_s}{r_1} =$

$(P_s)_n = P_{s1}\left(\frac{r_n}{r_1}\right)$

$\boxed{\frac{P_{s1}}{r_1}[r_1^2 + r_2^2 + r_3^2 + r_4^2] = P \times e}$

$P_{s1} = ?$

(8) $(\theta_1 = \theta_2) < (\theta_3 = \theta_4)$

$(R_1 = R_2) > (R_3 = R_4)$

$R_{max} = R_1 = R_2$

(9) $R_{max} = \sqrt{P_p^2 + P_{s_1}^2 + 2P_pP_{s_1}\cos\theta_1}$

$= \_\_\_\_\_\_ N$

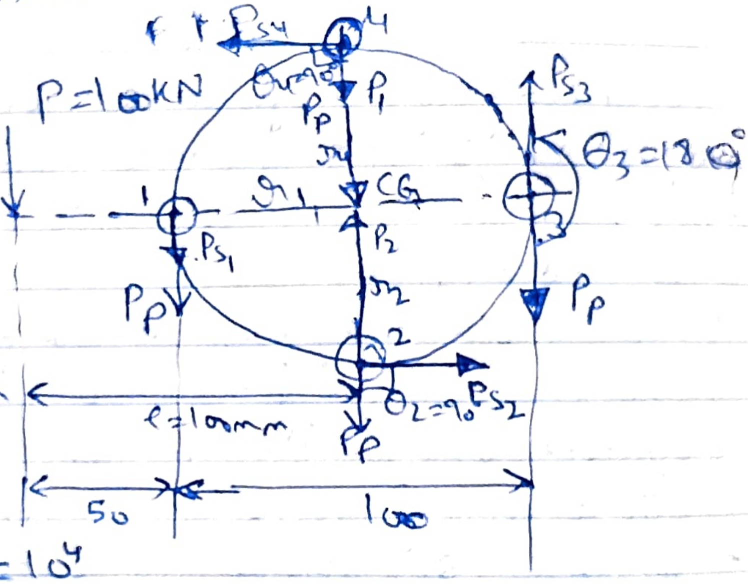
(10) dia of rivets (d)

$(\tau_{max})_{ind} \le \tau_{per}$

$\frac{R_{max}}{\pi/4\, d^2} \le \tau_{per} \Rightarrow d \ge \_\_\_$

For an eccentrically loaded riveted joint as shown in the figure, determine diameter of the rivets if permissible shear stress = 60 MPa

Sol^n:



$P_p = \frac{P}{4} = 25 kN$

T.M. = 100 × 100

$= 10^4$ kN-mm

(C.W.)

$r_1 = r_2 = r_3 = r_4 = 50$ mm

$P_{s_1} = P_{s_2} = P_{s_3} = P_{s_4}$

$\left[\frac{P_{s_1}}{r_1}(4r_1^2)\right] = 10^4$

$P_{s_1} \times 4 \times 50 = 10^4$

$\boxed{P_{s_1} = 50 kN}$

$(\theta_1 = 0) < (\theta_2 = \theta_4 = 90°) < (\theta_3 = 180°)$

$R_1 > (R_2 = R_4) > R_3$

$R_{max} = R_1 = P_p + P_{s_1}$

$= 75 kN$

$\tau_{max} \le \tau_{per}$

$\frac{4R_{max}}{\pi d^2} \le 60$

$d \ge 39.8$

$\boxed{d = 40 mm}$

$$R_2 = R_4 = \sqrt{P_p^2 + P_s^2}$$

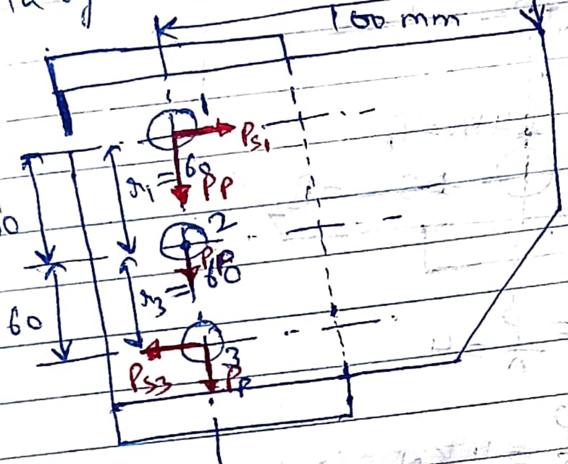
$$R_3 = P_{s3} - P_{s1} = 25 \text{ kN}$$

If all the rivets are equidistant from the CG of group of rivets, then the worst rivets are those rivets which are nearer to line of action of load.

For an eccentrically loaded riveted joint, determine the following:

(1) Worst loaded rivets.

(2) Shear stress induced in all the rivets if dia of rivets = 30 mm



$(r_1 = r_3 = 60) > (r_2 = 0)$

$(P_{s1} = P_{s3}) > (P_{s2} = 0)$

$\theta_1 = \theta_3 = 90°$

$R_{max} = R_1 = R_3 = \sqrt{P_p^2 + P_{s1}^2}$

$R_2 = P_p$

$$P_p = P/3 = 20 \text{ kN}$$

$$\frac{P_{s1}}{r_1}\left[2r_1^2 + r_2^2\right] = 60 \times 100$$

$$P_{s1} \times 2 \times 60 = 60 \times 100$$

$$P_{s1} = 50 \text{ kN}$$

$$R_{max} = \sqrt{(20)^2 + (50)^2}$$
$$= 53.85 \text{ kN}$$

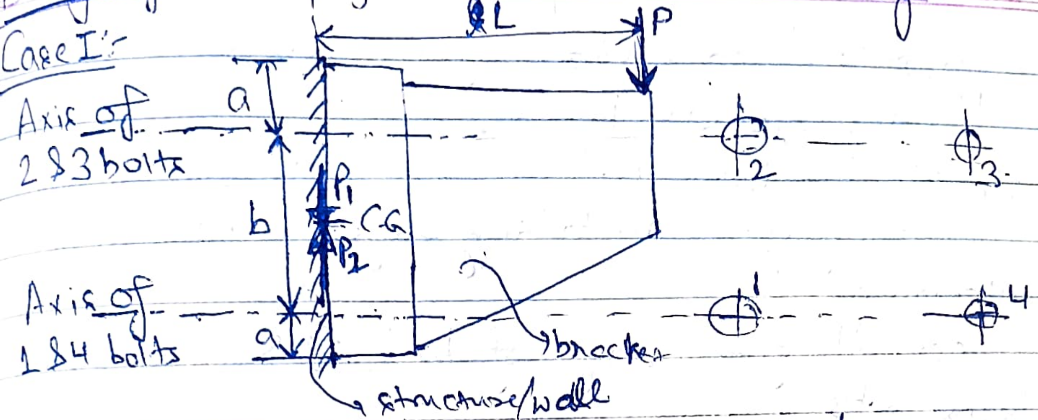
$$\tau_{max} = \frac{4 R_{max}}{\pi d^2} = \frac{4 \times 53.85 \times 10^3}{\pi (30)^2} = 76.18 \text{ MPa}$$

$$R_2 = P_p = 20 \text{ kN}$$

$$\tau_2 = \frac{4 \times 20 \times 10^3}{\pi (30)^2} = 28.29 \text{ MPa}$$

Design of bolted joint under eccentric loading

Case I :-



Load is acting $\perp^{ar}$ to axis of bolts/parallel to plane of bolts (bolts are subjected to shear & tensile stresses). {butt joint}

(1) Introduce two equal & opposite forces P_1 & P_2 through the C.G. of group of bolts as shown in figure & $P_1 = P_2 = P$

(2) Eccentricity, $e = L$

(3) Effect of P_1:- is to cause a shear stress of equal magnitude at each & every bolt.

$$(P_s)_{bolt} = \frac{P_1}{n} = \frac{P}{4}$$

$$(\tau_s)_{bolt} = \frac{(P_s)_{bolt}}{\pi/_4 d_c^2}$$

$$(\tau_s)_{bolt} = \frac{P}{\pi d_c^2} = \frac{x}{d_c^2} \text{ MPa} \quad \text{—} (I)$$

where d_c = core or minor diameter of bolts in mm

(4) Effect of P & P_2

P & P_2 causes a moment of $(P \times L)$ in the CW direc^n.

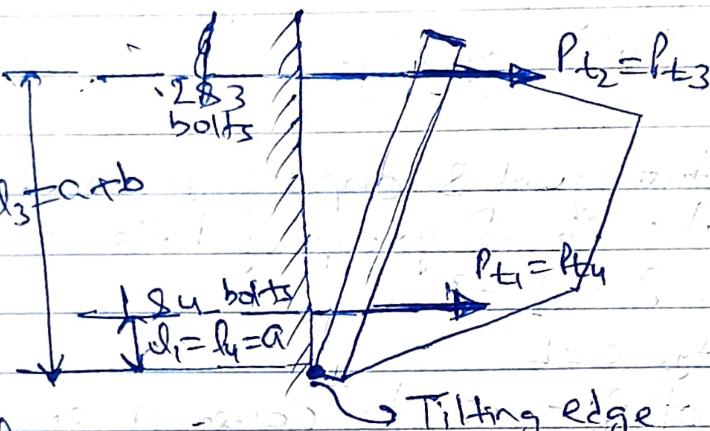
$$M = P \times e = P \times L \ (\circlearrowright)$$

Due to this moment, the bolts are subjected to tensile forces P_t when the bracket is tilted about the bottom edge as shown in the figure.

(5) Tensile force developed in a bolt is directly proportional to distance of its axis from the tilting edge. (i.e. $P_T \propto l$).

(6) Hence the worst bolt are those bolts which are far away from the tilting edge (i.e. where l is maximum)

For the given arrgt of bolts, ② & ③ bolts are worst bolts.



(7) l_{max} :-

$$(l_2 = l_3 = a+b) > (l_1 = l_4 = a)$$

$$(P_{t_2} = P_{t_3}) > (P_{t_1} = P_{t_4}) \quad [\because P_t \propto l]$$

$$(P_t)_{max} = (P_{t_2}) = P_{t_3}$$

$P_{t_1} \propto l_1$

$P_{t_1} = k l_1$

$P_{t_2} = k l_2$ - - - P_{t_4} ...

(8) $(P_t)_{max}$

$$P_{t_1} l_1 + \text{- - - -} + P_{t_4} l_4 = P \times L$$

$$\frac{P_{t_2}}{l_2}\left[l_1^2 + l_2^2 + \text{- - -} + l_4^2\right] = P \times L$$

$$P_{t_2} = \text{———} N$$

(9) $$(\sigma_t)_{max} = \frac{4 (P_t)_{max}}{\pi d_c^2} = \frac{Y}{d_c^2} \text{ MPa} \quad \text{— ②}$$

(10) diameter of bolt (d)

MSST, $\tau_{per} = \frac{S_{ys}}{N} = \frac{1}{2}\sqrt{(\sigma_t)^2_{max} + 4(\tau_s)^2_{max}}$

$$\tau_{per} = \frac{1}{2}\sqrt{\left(\frac{Y}{d_c^2}\right)^2 + 4\left(\frac{X'}{d_c^2}\right)^2}$$

$d_c =$ ______ mm

$d_r = d = \frac{d_c}{0.84} = \underline{20.2}$ mm

M 22 bolts

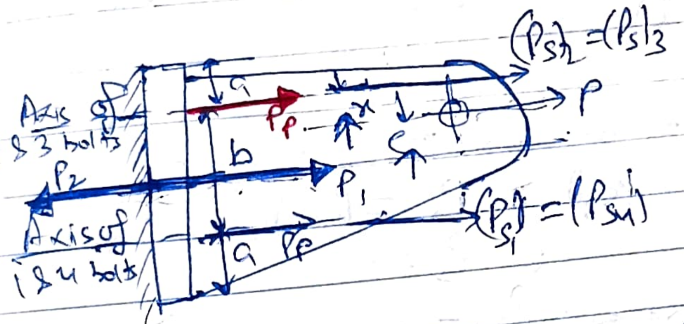
M → metric threads (thread profile) (V-threads)

22 → Major diameter

Case II :- Load is acting ⊥ to axis of bolts
Or load is acting ⊥ar to plane of bolts

⇓

bolts are subjected to primary & 2^nd ary tensile stress



(1) Determination of CG of group of bolts.

(2) Introduce two equal & opposite forces through the CG of group of holes. as shown in fig &

$P_1 = P_2 = P$

(3) Eccentricity $e = \frac{b}{2} - X$

(4) Effect of P_1 :- is to cause a primary tensile force of equal magnitude (P_p) at each & every bolt as shown in fig.

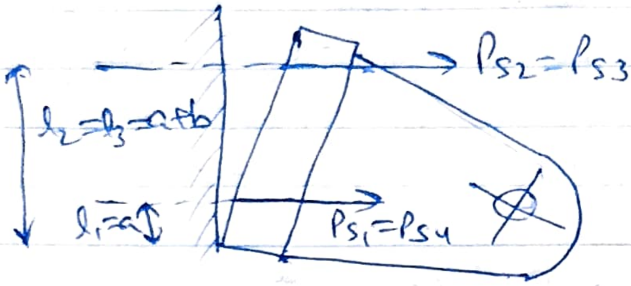
$P_p = \frac{P_1}{n} = P/4$

(5) Effect of P & P_2 :- Causes a moment of $(P \times e)$ in the CW direction.

$M = P \times e$ (↻)

Due to this moment, the bolts are subjected to

2ndary tensile force (P_s) & the bracket is tilted about the bottom edge as shown in figure.



- The secondary tensile force is maximum at the bolt which is far away from the tilting edge (because $P_s \propto l$)

(6) $(l_{max} = l_2 = l_3 = a+b) > (l_1 = l_4 = a)$

$$(P_{s2} = P_{s3}) > (P_{s1} = P_{s4})$$

$$(\because P_s \propto l)$$

$$(P_s)_{max} = P_{s2} = P_{s3}$$

(7) <u>$(P_s)_{max}$:-</u>

$$\frac{(P_s)_2}{l_2}\left[2l_1^2 + 2l_2^2\right] = P \times e$$

$$(P_s)_2 = \_\_\_\_\_\_$$

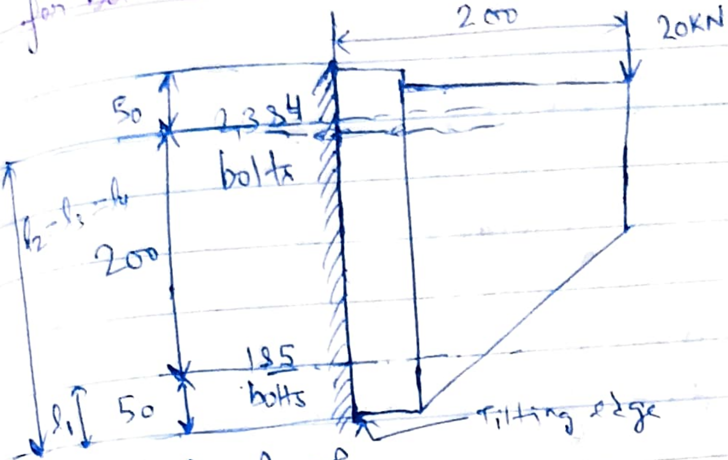
(8) $R_{max} = R_2 = R_3 = P_p + (P_s)_{max}$

(9) $(\sigma_t)_{max} \leq (\sigma_t)_{per}$; $\frac{4R_{max}}{\pi d_c^2} \leq (\sigma_t)_{per}$

$$d_c \geq \_\_\_\_ mm$$

$$d_n = \frac{d_c}{0.84} = \_\_\_\_ mm$$

A bracket is fitted to a vertical channel with 3 bolts at top & 2 bolts at bottom as shown in fig. Determine the diameter of bolt if yield strength in shear for bolt material is 150 MPa & FOS is equal to 4



Soln: (1) $P_1 = P_2 = P$

(2) $e = 200\text{mm}$

(3) Effect of P_1

$$(P_s)_{bolt} = \frac{P}{5} = \frac{20}{5} = 4\text{ KN}$$

$$(\tau_s)_{bolt} = \frac{P_s}{A_c} = \frac{4P_s}{\pi d_c^2}$$

$$(\tau_s)_{bolt} = \frac{4 \times 4000}{\pi d_c^2}$$

$$\tau_s = \frac{5.093 \times 10^3}{d_c^2}\text{ MPa} \quad — (1)$$

(4) Effect of P & P_2

$$M = P \times e = 20 \times 200$$

$$M = 4000\text{ KN-mm} \quad (2)$$

(5) $l_1 = l_5 = 50\text{mm}$, $l_2 = l_3 = l_4 = 250\text{mm}$

$$(l_2 = l_3 = l_4) > (l_1 = l_5)$$

$$(P_{t_2} = P_{t_3} = P_{t_4}) > (P_{t_1} = P_{t_5})$$

$$(P_t)_{max} = P_{t_2} = P_{t_3} = P_{t_4}$$

(6) $(P_t)_{max}$

$$P_{t_1} l_1 + \ldots + P_{t_5} l_5 = 4000$$

$$\frac{P_{t_2}}{l_2}\left[2l_1^2 + 3l_2^2\right] = 4000$$

$$P_{t_2} = 5.194\text{ KN}$$

(7) $(\sigma_t)_{max} = \frac{4(P_t)_{max}}{\pi d_c^2} = \frac{6.613 \times 10^3}{d_c^2}$ MPa

(8) by using MSST,

$$\tau_{per} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

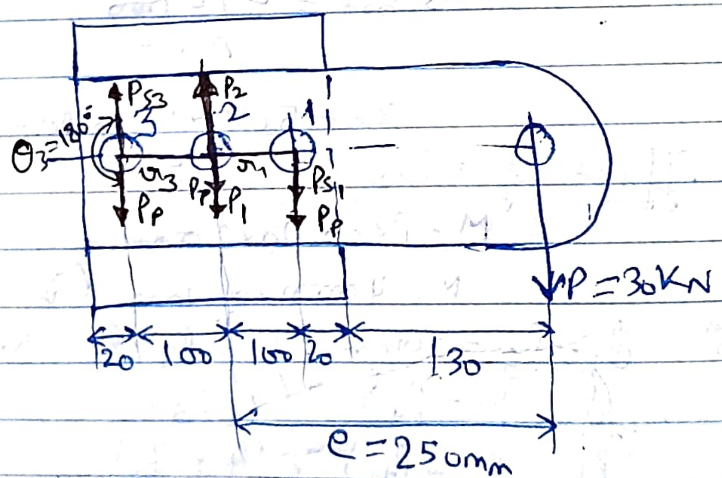
$$\frac{S_{ys}}{N} = \frac{1}{2}\sqrt{(\sigma_t)^2_{max} + 4\tau_s^2}$$

$$d = 12.7\,mm$$

$$d_n = \frac{d_c}{0.84} = 15.1\,mm$$

choose M 16 bolts.

For an eccentrically loaded bolted joint as shown in the figure, determine diameter of the bolts if $S_{yt} = 200$ MPa & FOS = 2. Also determine the resultant forces on all the bolts.



Sol^n: $P_p = \frac{P}{3} = 10$ KN

$$T.M. = P \times e = 30 \times 250 = 7500\,KN\text{-}mm$$

$$r_1 = r_3 = 100\,;\ r_2 = 0$$

$$(P_{s1} = P_{s3}) > (P_{s2} = 0)$$

$$\frac{P_{s1}}{r_1}\left[2r_1^2 + r_2^2\right] = 7500$$

$P_{s_1} \times 2 \times 100 = 7500$

$\boxed{P_{s_1} = 37.5KN}$

$(\theta_3 = 0°) < (\theta_3 = 180°)$

(1) bolt is worst

$R_{max} = R = P_p + P_{s_1} = 47.5KN$

$R_2 = P_p = 10KN$

$R_3 = P_{s3} - P_p = 27.5KN$

for safe design of bolts,

$$\frac{4R_{max}}{\pi d_c^2} \leq \tau_{per} \text{ or } \frac{S_{yt}}{2N}$$

$$\frac{4 \times 4.75 \times 10^3}{\pi d_c^2} \leq \frac{200}{2 \times 2}$$

$d_c \geq 34.7\,mm$

$d_n = \frac{d_c}{0.84} = 41.4\,mm$

M 42 bolts.

$(\tau)_{per} = 100MPa$

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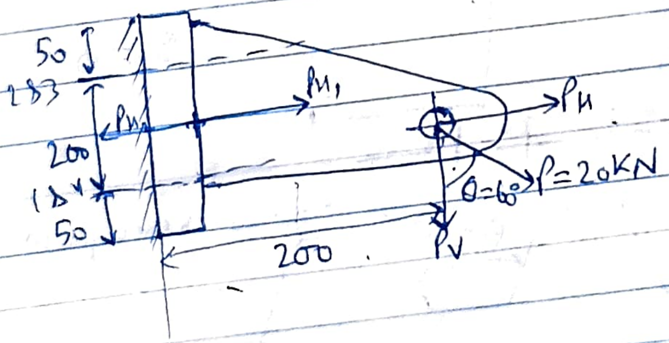

M_V
M_H
M_R
$(\tau)_s$